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Assignment:

1. Find the Integral of the following:

1.) $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Solution

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

When $x-1=0$, then $x=1$

$$3(1)-1 = A(1-2)(1-3) + B(0)(x-3) + C(0)(x-2)$$

$$2 = A(-1)(-2) + B(0) + C(0)$$

$$2 = A(2)$$

$$A = 2/2 = 1$$

when $x-2=0$

$$x = 2$$

$$3(2)-1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$$

$$6-1 = A(0) + B(1)(-1) + C(0)$$

$$5 = B(-1)$$

$$B = 5/-1 = -5$$

when $x-3=0$, $x=3$

$3(3)-1 = A(3-2)(3-3)+B(3-1)(3-3)+C(3-1)(3-2)$

$8 = A(0)+B(0)+C(2)(1)$

$8 = 2C$

$C = 8/2 = 4$

$$\frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \frac{1}{x-1}+\frac{-5}{x-2}+\frac{4}{x-3}$$

$$\therefore \int 1/x-1 + \int -5/x-2 + \int 4/x-3$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3$$

2. $$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)}$$ note $x^2+1 = x^2+1^2 = (x+1)$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2}+\frac{B}{x^2+1}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+B(x+2)}{(x+2)(x^2+1)}$$

$x^2+x+1 = A(x^2+1)+B(x+2)$

when $x+2=0$ then $x=-2$

$(-2)^2+(-2)+1 = A((-2)^2+1)+B(-2+2)$

$4-2+1 = A(5)+B(0)$

$3 = A(5)$

$A = 3/5$

3) $\int \frac{x^2+1}{(x-3)(x-2)^2}$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$$

$x^2+1 = A(x-2)^2 + B(x-2)(x-3) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-3x-2x+6) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$x^2+1 = x^2(A+B) + x(-4A-5B+C) + (4A+6B-3C)$

$A+B = 1 \quad --- (i)$

$-4A-5B+C = 0 \quad --- (2)$

$4A+6B-3C = 1 \quad --- (3)$

$0 = -4A-5B+C$

$-C = -4A-5B$

$C = 4A+5B \quad --- (iv)$

$1 = 4A+6B-3C$

$1 = 4A+3B-3(4A+5B)$

$1 = 4A+3B-12A-15B$

$1 = -8A-12B \quad --- (ii)$

$1 = A+B \times -8$

$-8 = -8A-8B$

$-1 = -8A-12B$

$-9 = 0+4B$

$-9 = 4B$

$B = -9/4$

$1 = A+B$

$1 = A-9/4$

$A = {}^{1+9}/_{4}$

$A = {}^{13}/_{4}$

$C = 4A + 5B$

$C = 4({}^{13}/_{4}) + 5({}^{9}/_{4})$

$C = 13 - {}^{45}/_{4}$

$C = {}^{7}/_{4}$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{13/4}{(x-3)} + \frac{-9}{4(x-2)} + \frac{7}{4(x-2)}$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{13}{4(x-3)} - \frac{9}{4(x-2)} + \frac{7}{4(x-2)}\,dx$$

$$= \frac{13}{4}\ln(x-3) - \frac{9}{4}\ln(x-2) + \int \frac{7}{4(x-2)}\,dx$$

$$= \frac{7}{4}\int \frac{1}{u^2}\,dx \qquad u = x-2$$

$$du/dx = 1\,dx = 1\,du$$

$$\frac{7}{4}\int u^{-2}\,du$$

$$\frac{7}{4}\,\frac{u^{-2+1}}{-2+1} + c = \frac{7u^{-1}}{-1} + c$$

$$-\frac{7}{4}u^{-1} + c = \frac{7}{4}(x-2)^{-1} + c$$

$$= 13/4\ln(x-3) - 9/4\sin(x-2) + 7/4(x-2)^{-1} + c.$$

4) $\int \frac{x^3 + x^2 + x + 1}{x-1} \, dx$

sln

$\int \frac{x^3 + x^2 + x + 1}{x-1} \, dx$

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x-1 & x^3 + x^2 + x + 1 \\ & -x^3 - x^2 \\ \hline & 2x^2 + x + 1 \\ & 2x^2 - 2x \\ \hline & 3x + 1 \\ & 3x - 3 \\ \hline & 4 \end{array}$$

$$\frac{x^3 + x^2 + x + 1}{x-1} = x^2 + 2x + 3 + \frac{4}{x-1}$$

$$= \int x^2 + 2x + 3 + \frac{4}{x-1} \, dx$$

$$= \frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4\ln(x-1) + c$$

$$= x^3/3 + x^2 + 3x + 4\ln(x-1) + c$$