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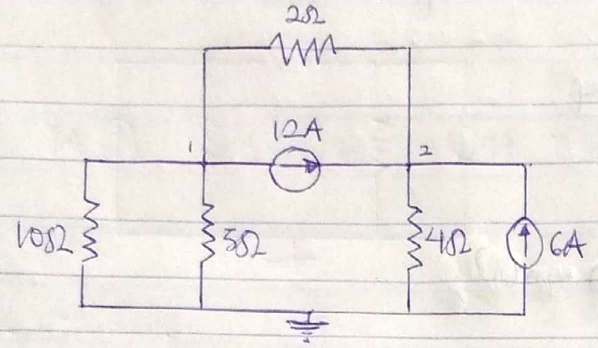
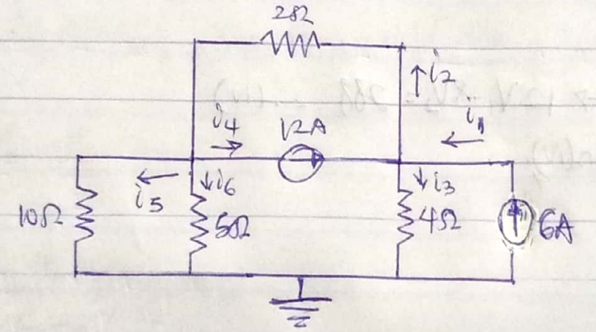
① Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below:





Applying KCL to node 1, $i_5 + i_6 + i_4 = i_2$

$i_5 = \frac{V_2 - 0}{10}$, $i_6 = \frac{V_2 - 0}{5}$, $i_4 = 12$, $i_2 = \frac{V_1 - V_2}{2}$

$$\frac{V_2}{10} + \frac{V_2}{5} + \frac{12}{1} = \frac{V_1 - V_2}{2} \quad \ldots (i)$$

Multiplying eqn (i) through by 10

$V_2 + 2V_2 + 120 = 5V_1 - 5V_2$

$\therefore V_2 + 2V_2 + 5V_2 - 5V_1 = -120$

$8V_2 - 5V_1 = -120$

$5V_1 - 8V_2 = 120 \quad \ldots (ii)$

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Applying KCL at node 2, $i_1 + i_4 = i_2 + i_3$

$i_1 = 6A$, $i_4 = 12$, $i_2 = \frac{V_1 - V_2}{2}$, $i_3 = \frac{V_1 - 0}{4}$

$$6 + 12 = \frac{V_1 - V_2}{2} + \frac{V_1}{4}$$

$$18 = \frac{V_1 - V_2}{2} + \frac{V_1}{4}$$

Multiplying through by 4

$$72 = 2V_1 - 2V_2 + V_1$$

$$72 = 3V_1 - 2V_2 \quad \ldots \text{(iii)}$$

[illegible]

$$8V_1 - 8V_2 = 120$$

$$3V_1 - 2V_2 = 72$$

Multiplying eqn (iii) by 4 $\rightarrow$ $12V_1 - 8V_2 = 288 \quad \ldots \text{(iv)}$

Substracting eqn (ii) from eqn (iv),

$$\begin{array}{r} 12V_1 - 8V_2 = 288 \\ 8V_1 - 8V_2 = 120 \\ \hline 7V_1 \quad = 168 \end{array}$$

$\therefore V_1 = 24V$

Substituting V_1 into eqn (iii)

$$3(24) - 2V_2 = 72$$

$$72 - 2V_2 = 72$$

$$72 - 72 = 2V_2$$

$$0 = 2V_2$$

$\therefore V_2 = 0V$

The currents flowing through the four resistors, (i_2, i_3, i_5 and i_6),

$$i_2 = \frac{V_1 - V_2}{2} = \frac{24 - 0}{2} = \frac{24}{2} = 12\ A$$

$$i_3 = \frac{V_1}{4} = \frac{24}{4} = 6\ A$$

$$i_5 = \frac{V_2}{10} = \frac{0}{10} = 0A,$$

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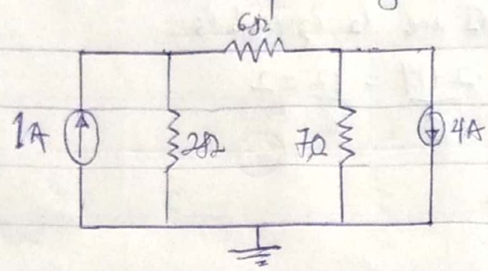
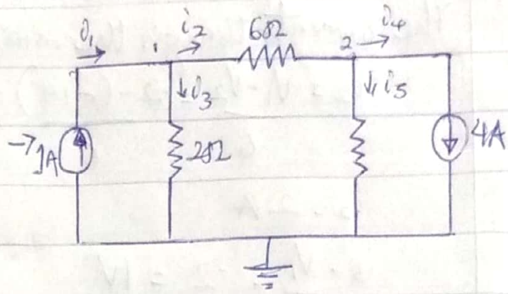
$i_6 = \frac{V_2}{5} = \frac{0}{5} = 0A$

The currents flowing through the four resistors are $i_2 = 12A$, $i_3 = 6A$, $i_5 = 0A$ and $i_6 = 0A$.

(ii) Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii), if the 2A current source was replaced by a 1A current source





Applying KCL to node 1,

$i_1 = i_2 + i_3$; $i_1 = 1A$, $i_2 = \frac{V_1 - V_2}{6}$, $i_3 = \frac{V_1}{2}$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

Multiplying through by 6

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ ... (i)

Applying KCL to node 2, $i_2 = i_4 + i_5$

$i_4 = 4$; $i_5 = V_2/7$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

Multiplying through by 42

$7V_1 - 7V_2 = 168 + 6V_2$

$7V_1 - 13V_2 = 168$ ... (ii)

From eqn (i), $V_2 = 4V_1 - 6$

Substituting V_2 in eqn (ii)

$7V_1 - 13(4V_1 - 6) = 168$

$7V_1 - 52V_1 + 78 = 168$

$-45V_1 = 90$

$V_1 = -2V$

Substituting $-2V$ for V_1 in eqn (i)

$6 = 4(-2) - V_2$

$6 = 8 - V_2$

$-V_2 = 14$

$V_2 = -14V$

The currents through the resistors are i_2, i_3 and i_5

6Ω: $i_2 = \frac{V_1 - V_2}{6} = \frac{-2-(-14)}{6} = \frac{-2+14}{6} = \frac{12}{6} = 2$

$i_2 = 2A$

2Ω: $i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_3 = -1A$

7Ω: $i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2$

$i_5 = -2A$

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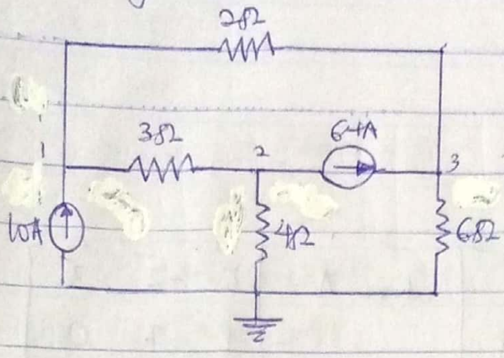
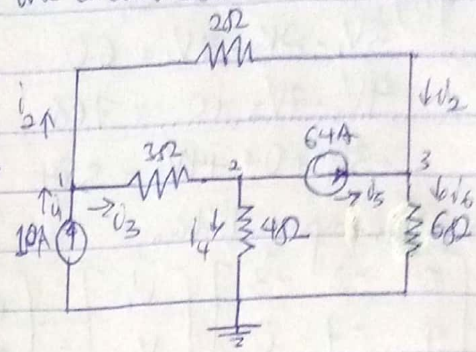
(2) Find the voltages at nodes 1, 2 and 3 in the circuit below:



$\rightarrow$



Applying KCL to node 1, $i_1 = i_2 + i_3$

$i_1 = 10A$, $i_2 = \frac{V_1 - V_3}{2}$, $i_3 = \frac{V_1 - V_2}{3}$, $i_4 = \frac{V_2}{4}$, $i_5 = 64$, $i_6 = \frac{V_3}{6}$

$$10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

Multiplying through by 6

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \ldots (i)$$

Applying KCL to node 2, $i_3 = i_4 + i_5$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{4} + \frac{64}{1}$$

Multiplying through by 12

$$4V_1 - 4V_2 = 3V_2 + 768$$

$$4V_1 - 7V_2 = 768 \quad \ldots (ii)$$

Applying KCL to node 3, $i_5 + i_2 = i_6$

$$64 + \frac{V_1 - V_3}{2} = \frac{V_3}{6}$$

Multiplying through by 6

$$384 + 3V_1 - 3V_3 = V_3$$

$$-3V_1 + 4V_3 = 384 \quad \ldots (iii)$$

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Applying Cramer's Rule,

$$5V_1 - 2V_2 - 3V_3 = 60$$
$$4V_1 - 7V_2 + 0 = 768$$
$$-3V_1 + 0 + 4V_3 = 384$$

In matrix representation,

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix} \quad \text{where } \Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$$5\begin{vmatrix} -7 & 0 \\ 0 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 4 & 0 \\ -3 & 4 \end{vmatrix} - 3\begin{vmatrix} 4 & 0 \\ -3 & 4 \end{vmatrix} = 5(-28) + 2(16) - 3(-21)$$
$$= -140 + 32 + 63 = -45$$

$$\Delta = -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60\begin{vmatrix} -7 & 0 \\ 0 & 4 \end{vmatrix} + 2\begin{vmatrix} 768 & 0 \\ 384 & 4 \end{vmatrix} - 3\begin{vmatrix} 768 & -7 \\ 384 & 0 \end{vmatrix}$$
$$= 60(-28) + 2(3072) - 3(2688)$$
$$= -1680 + 6144 - 8064 = -3600$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$$

$$V_1 = 80V$$

$$\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix} = 5\begin{vmatrix} 768 & 0 \\ 384 & 4 \end{vmatrix} - 60\begin{vmatrix} 4 & 0 \\ -3 & 4 \end{vmatrix} - 3\begin{vmatrix} 4 & 768 \\ -3 & 384 \end{vmatrix}$$
$$= 5(3072) - 60(16) - 3(1536 + 2304)$$
$$= 15360 - 960 - 3(3840)$$
$$= 15360 - 960 - 11520$$
$$\Delta_2 = 2880$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

$$V_2 = -64V$$

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$$\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5\begin{vmatrix} -7 & 768 \\ 0 & 384 \end{vmatrix} + 2\begin{vmatrix} 4 & 768 \\ -3 & 384 \end{vmatrix} + 60\begin{vmatrix} 4 & -7 \\ -3 & 0 \end{vmatrix}$$

$$= 5(-2688) + 2(1536 + 2304) + 60(-21)$$

$$= -13440 + 2(3840) - 1260$$

$$= -13440 + 7680 - 1260$$

$$\Delta_3 = -7020$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$$

$$V_3 = 156V$$

The voltages at the nodes 1, 2 and 3 are

$V_1 = 80V$

$V_2 = -64V$

$V_3 = 156V$

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