YILKUDI RITJI SIMON

① $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

multiply through by $(x-1)(x-2)(x-3)$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$

$= Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 5B + Cx^2 - 3Cx + 2C$

$3x-1 = (A+B+C)x^2 + (-5A-4B-3C)x + 6A + 3B + 2C$

$A + B + C = 0 \quad 1$

$-5A - 4B - 3C = 3 \quad 2$

$6A + 3B + 2C = -1 \quad 3$

From 1

~~A~~ $A = -B - C$

Put A in 2 and 3

$-5(-B-C) - 4B - 3C = 3$

$6(-B-C) + 3B + 2C = -1$

$B + 2C = 3$ $\times 3$

$-3B - 4C = -1$ $\times 1$

$+ \quad 3B + 6C = 7$

$-3B + 4C = -1$

$\overline{\qquad\qquad 2C = 8}$

$C = 4$

$B + 2C = 3$

$B + 2(4) = 3$

$B + 8 = 3$

$B = 3 - 8$

$B = -5$

$A = -B - C$

$= -(-5) - 4$

$= 1$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{1}{x-1}\,dx + \int \frac{-5}{x-2}\,dx + \int \frac{4}{x-3}\,dx$$

$$\ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

(2) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)+(x+2)}{(x+2)(x^2+1)}$$

multiply through by $(x+2)(x^2+1)$

$$x^2+x+1 = A(x^2+1)+(Bx+C)(x+2)$$

$$x^2+x+1 = Ax^2+A+Bx^2+2Bx+(Cx+2C)$$

$$x^2+x+1 = Ax^2+Bx^2+2Bx+Cx+A+2C$$

$$x^2+x+1 = x^2(A+B)+x(2B+C)+A+2C$$

$A+B=1$ ①

$2B+C=1$ ②

$A+2C=1$ ③

From ① $A = 1-B$

Put, ~~$1-B$~~ $1-B$ for A and C

$A = 1 - \frac{2}{5}$

$(1 - B) + 2C = 1$

$1 - B + 2C = 1$

$-B + 2C = 1 - 1$

$-B + 2C = 0$

$2B + C = 1 \quad \times 1$

$-B + 2C = 0 \quad \times 2$

$2B + C = 1$

$-2B + 4C = 0$

$5C = 1$

$C = \frac{1}{5}$

$2B + C = 1$

$2B + \frac{1}{5} = 1$

$2B = 1 - \frac{1}{5}$

$2B = \frac{4}{5}$

$B = \frac{4}{5} \div 2$

$B = \frac{2}{5}$

$\int \frac{dx}{5(x+4)}$

$A = 1 - \frac{2}{5}$

$A = \frac{3}{5}$

$$\int \frac{x^2 + x + 1}{(x+2)(x^2+2)} = \int \frac{\frac{3}{5}}{x+2} dx + \int \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1}$$

$$= \int \frac{\frac{3}{5}}{(x+2)} dx + \int \frac{\frac{2}{5}}{(x^2+1)} + \int \frac{\frac{1}{5}}{x^2+1}$$

$$= \int \frac{3}{5(x+2)} dx + \int \frac{2}{5(x^2+1)} dx + \int \frac{dx}{5(x^2+4)}$$

$$= \frac{3}{5} \int \frac{dx}{5(x+2)} + \frac{2}{5} \int \frac{dx}{5(x^2+1)} + \frac{1}{5} \int \frac{dx}{5(x^2+1)}$$

let $v = x^2 + 1$

$$\frac{dv}{dx} = 2x$$

$$dv = 2x\,dx$$

$$x\,dx = \frac{dv}{2}$$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x-3$

$\frac{3}{5}\int \frac{dx}{(x+2)} + \frac{2}{5} + \frac{1}{2}\int \frac{du}{u} + \frac{1}{5}\int \frac{dx}{(x^2+1)}$ 3

$\frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{du}{u} + \frac{1}{5}$ are tan x

$-3x$

$\frac{3}{5}\ln(x+1) + \frac{1}{5}\ln(x^2+1) + \frac{1}{5}$ are tan $x + c$

③ $\frac{x^2+1}{(x-3)(x-2)^2}$

$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ →

$\frac{x^2+1}{(x-3)(x-2)} = \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$

multiply throughs by $(x-3)(x-2)^2$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2-4Ax+4A+Bx^2-5Bx+6B$
$+xC-3C$

$= Ax^2+Bx^2-4Ax-5Bx+xC+4A+6B-3C$

$= x^2(A+B)+B(-4A-5B+C)+(4A+6B-3C$

$A+B=1$ (1)

$-4A-5B+C=0$ (2)

$4A+6B-3C=1$ (3)

$A=1-B$

Put (4) in (2) and (3):

$-4(1-B)-5B+C=0$

$-4+4B-5B+C=0$

$-4-B+C=0$

$-B+C=4$

$4(1-B)+6B-3C=1$

$4-4B+6B-3C=1$

$4+2B-3C=1$

~~$2B-3C=+3$~~

$2B - 3C = -3$

$2B - 3C = -3 \quad \times 1$

$-B + C = 4 \quad \times 2$

$2B - 3C = -3$

$-2B + 2C = 8$

$-C = 5$

$C = -5$

$2B - 3C = -3$

$2B - 3(-5) = -3$

$2B + 15 = -3$

$2B = -18$

$B = -9$

$A = 1 - B$

$= 1 - (-9)$

$= 10$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{10}{x-3}\,dx + \int \frac{-9}{x-2}\,dx + \int \frac{-5}{(x-2)^2}\,dx$$

$$= \int \frac{10}{x-3}\,dx - \int \frac{9}{x-2}\,dx - \int \frac{5}{(x-2)^2}\,dx$$

$$\cdot\ 10\ln(x-3) - 9\ln(x-2) + 5(x+2)^{-1} + C$$

(4) $$\int \frac{x^3+x^2+x+1}{x-1}\,dx$$

$$\begin{array}{r|l} x-1 & x^2+2x+3 \\ \hline -(x^3+x^2+x+1) & \\ x^3-x^2 & \\ \hline & 2x^2+x+1 \\ & -(2x^2-2x) \\ \hline & 3x-1 \\ & 3x-3 \\ \hline & 4 \end{array}$$

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx = \int x^2+2x+3\,dx + \int \frac{4}{x-1}\,dx$$

$$\frac{x^3}{3} + \frac{3x^2}{2} + 3x + 4\ln(x-1) + C$$

$$\frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$