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17/SCI01/027
MAT204 ASSIGNMENT

Let $A = \begin{Bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{Bmatrix}$ $\quad B = \begin{Bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{Bmatrix}$

$C = \begin{Bmatrix} 2 & 1 & 3 \\ 2 & 2 & 4 \\ 0 & 1 & 1 \end{Bmatrix}$

1. Linear transformation of A if vector X = (a, b, c)

<u>solution</u>

$$A = \begin{Bmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{Bmatrix}, \quad X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 3a \\ a \\ 2a \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ b \end{pmatrix} + \begin{pmatrix} 2c \\ 2c \\ c \end{pmatrix}$$

$$T(x) = \begin{Bmatrix} 3a + b + 2c \\ a + 0 + 2c \\ 2a + b + c \end{Bmatrix}$$

Hence the transformation of

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \text{gives} \quad \begin{Bmatrix} 3a + b + 2c \\ a + 0 + 2c \\ 2a + b + c \end{Bmatrix}$$

2. Find the rank of (B+C) transpose

$$B+C = \begin{Bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{Bmatrix} + \begin{Bmatrix} 2 & 1 & 3 \\ 2 & 2 & 4 \\ 0 & 1 & 1 \end{Bmatrix}$$

$$B+C = \begin{Bmatrix} 3 & 1 & 5 \\ 4 & 2 & 5 \\ 1 & 2 & 4 \end{Bmatrix}$$

$$(B+C)^T = \begin{Bmatrix} 3 & 4 & 1 \\ 1 & 2 & 2 \\ 5 & 5 & 4 \end{Bmatrix}$$

To find rank

$$|(B+C)^T| = 3\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix} - 4\begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 5 & 5 \end{vmatrix}$$

$$= 3(8 - 10) – 4(4 - 10) + 1(5 - 10)$$
$$= 3(-2) – 4(-6) + 1(5 – 10)$$
$$= -6 + 24 - 6$$
$$= 12$$
$$12 \neq 0$$

Hence the Rank of $(B+C)^T$ is 3.

3. Check whether A, B, and C are singular or non-singular matrix.

For A;

$$|A| = \begin{vmatrix} 3 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix}$$

$$|A| = 3\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 3(0 – 2) – 1(1 – 4) + 2(1 – 0)$$
$$=3(-2) -1(-3) + 2(1)$$
$$= -6 + 3 + 2 = -1$$
$$-1 \neq 0$$

∴ It is a non-singular matrices.

For B;

$$|B| = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$|B| = 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$$

$|B| = 1(0 - 1) - 0 + 2(2 - 0)$

$= 1(-1) - 0 + 2(2)$

$= -1 - 0 + 4$

$= 3$

$3 \neq 0$

$\therefore$It is a non-singular matrices.

For C;

$$|C| = \begin{vmatrix} 2 & 1 & 3 \\ 2 & 2 & 4 \\ 0 & 1 & 1 \end{vmatrix}$$

$$|C| = 2\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix}$$

$|C| = 2(2 - 4) - 1(2 - 0) + 3(2 - 0)$

$= 2(-2) - 1(2) + 3(2)$

$= -4 - 2 + 6$

$= 0$

$\therefore$It is a singular matrix.