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DEPT: AERONAUTICAL ENGINEERING

Course:- GENERAL MATHEMATICS III, MAT 104

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find the Integral of the following

a) $\frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Solution

$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$. Resolving $\frac{3x-1}{(x-1)(x-2)(x-3)}$ into partial fractions.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A((x-2)(x-3)) + B((x-1)(x-3)) + C((x-1)(x-2))}{(x-1)(x-2)(x-3)}$$

$\therefore 3x-1 = A(x(x-3)-2(x-3)) + B(x(x-3)-1(x-3))$

$+ C(x(x-2)-1(x-2)))$

$3x - 1 = A(x^2 - 5x + 6) + B(x^2 - 4x + 3) + C(x^2 - 3x + 2)$

$3x - 1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 + 3Cx + 2C$

Collect like terms on the RHS

$3x - 1 = Ax^2 + Bx^2 + Cx^2 - 5Ax - 4Bx - 3Cx + 6A + 3B + 2C$

$3x - 1 = x^2(A + B + C) - x(5A + 4B + 3C) + (6A + 3B + 2C)$

Comparing with LHS

$\therefore A + B + C = 0 \; --- (i)$

$-5A - 4B - 3C = 3 \; --- (ii)$

$6A + 3B + 2C = -1 \; --- (iii)$

from (i)

$A = -B - C \; --- (iv)$; Put (iv) into (ii)

$-5(-B - C) - 4B - 3C = 3$, $\therefore 5B + 5C - 4B - 3C = 3$,

$\therefore B + 2C = 3 \; --- (v)$; Put (iv) into (iii)

$6(-B - C) + 3B + 2C = -1$, $\therefore -6B - 6C + 3B + 2C = -1$

$-3B - 4C = -1$, $\therefore 3B + 4C = 1 \; --- (vi)$

from (v) form (vii)

$B + 2C = 3$, $\therefore B = 3 - 2C \; --- (vii)$; Put (vii) into (vi)

$\therefore 3B + 4C = 1, \quad 3(3 - 2C) + 4C = 1$

$9 - 6C + 4C = 1, \quad 9 - 2C = 1$

$2C = 9 - 1, \quad \frac{2C}{2} = \frac{8}{2} \quad \therefore C = \underline{\underline{4}}$

Put C into (ii) to get B

$\therefore B + 2C = 3, \quad B + 8 = 3, \quad B = 3 - 8 = \underline{\underline{-5}}$

from (i) i.e $A + B + C = 0$

$\therefore A - 5 + 4 = 0, \quad A - 1 = 0 \quad \therefore A = \underline{\underline{1}}$

$\therefore A = 1, \ B = -5, \ \text{and} \ C = 4$

$$\therefore \frac{3x - 1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\int \frac{3x - 1}{(x-1)(x-2)(x-3)} = \int \frac{1dx}{(x-1)} + \int \frac{-5}{(x-2)} dx + \int \frac{4}{(x-3)} dx$$

$$= \int \frac{dx}{(x-1)} - 5\int \frac{dx}{(x-2)} + 4\int \frac{dx}{(x-3)}$$

$$\therefore \int \frac{3x - 1}{(x-1)(x-2)(x-3)} = \underline{\underline{\ln(x-1) - 5\ln(x-5 + 4\ln(x-x) + C}}$$

ans.

b) $\frac{x^2+x+1}{(x+2)(x^2+1)} dx$

solution

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

$\therefore \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$, simplifying

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$\therefore x^2+x+1 = Ax^2+A+Bx(x+2)+C(x+2)$

$x^2+x+1 = Ax^2+A+Bx^2+2Bx+Cx+2C$

$x^2+x+1 = Ax^2+Bx^2+2Bx+Cx+A+2C.$

$x^2+x+1 = x^2(A+B)+x(2B+C)+(A+2C).$

$\therefore A+B=1$ ---(i), $2B+C=1$ -(ii), $A+2C=1$ ---(iii)

$A=1-B$, but $A=1-2C$ i.e from (i) + (iii)

$\therefore 1-B=1-2C$, $1-1-B+2C=0$, $2C-B=0$

$\therefore B=2C$. put $2C$ into eqn (ii).

EV1

$\therefore 2B + C = 1$; $2(2C) + C = 1$, $4C + C = 1$, $5C = 1$

$\therefore C = 1/5$ from (ii), $2B + C = 1$, $2B + 1/5 = 1$

$2B = \frac{1}{1} - \frac{1}{5}$, $\frac{2B}{2} = \frac{4}{5} \div 2$, $B = \frac{2}{5}$

$\therefore A + B = 1$, $A + \frac{2}{5} = 1$, $A = \frac{1}{1} - \frac{2}{5} = 3/5$

$\therefore A = 3/5$, $B = 2/5$, $C = 1/5$

$$\therefore \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3/5}{(x+2)} + \frac{2/5 x + 1/5}{(x^2+1)}$$

$$= 3/5\left(\frac{1}{x+2}\right) + 1/5\left(\frac{2x+1}{x^2+1}\right)$$

$$= 1/5\left[\left(\frac{3}{x+2}\right) + \left(\frac{2x+1}{x^2+1}\right)\right]$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{1}{5}\left[\int \frac{3}{x+2}\,dx + \int \frac{2x+1}{x^2+1}\,dx\right]$$

$$= \frac{1}{5}\left[3\int \frac{dx}{x+2} + \int \frac{2x+1}{x^2+1}\,dx\right]$$

Considering $\int \frac{2x+1}{x^2+1}\,dx$ rehane:

$\int \frac{2x+1}{x^2+1^2}$ $\therefore x = 1\tan\theta = \tan\theta$

$dx/d\theta = \sec^2\theta$, $dx = \sec^2\theta\, d\theta$

$\therefore x^2+1^2 = 1\tan^2\theta + 1^2 + 1(\tan^2\theta+1)$

Substituting, we have!

$$\int \frac{2x+1}{x^2+1^2} = \int \frac{\cancel{\sec^2\theta}\, d\theta\,(2x+1)}{\cancel{\sec^2\theta}} = \int d\theta\,(2x+1)$$

but $x = \tan\theta$,

$$\therefore \int d\theta\,(2x+1) = \int d\theta\,(2\tan\theta+1) = \int d\theta\, 2\tan\theta + d\theta$$

$$= \int 2\tan\theta\, d\theta + \int d\theta$$

$$= 2\int \tan\theta\, d\theta + \theta = 2(-\ln(\cos\theta)) + \theta$$

$\therefore -2\ln\cos\theta + \theta$. but $\theta = \tan^{-1}x$

$\therefore = -2\ln\cos(\tan^{-1}x) + \tan^{-1}x$, factorize

$\tan^{-1}x(-2\ln\cos 1 + 1) = \tan^{-1}x(1) = \tan^{-1}x$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{1}{5}\left[3\ln(x+2) + \tan^{-1}x\right] + C$$

ans.

( ) $(x^2+1)/(x-3)(x-2)^2\,dx$

soln

$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = ?$ by resolving to fractions.

P.T.O

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B[(x-3)(x-2)] + C(x-3)}{(x-3)(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A[x(x-2)-2(x-2)] + B[x(x-2)-3(x-2)] + Cx-3C}{(x-3)(x-2)^2}$$

$\therefore x^2+1 = A(x^2-4x+4)+B(x^2-5x+6)+Cx-3C$

$x^2+1 = Ax^2-4Ax+4A+Bx^2-5Bx+6B+Cx-3C$

$x^2+1 = Ax^2+Bx^2-4Ax-5Bx+Cx+4A+6B-3C$

$\therefore x^2+1 = x^2(A+B)-x(4A+5B-C)+(4A+6B-3C)$

Comparing, $A+B=1$ ---(i) $-4A-5B+C=0$ ---(ii)

$4A+6B-3C=1$ ---(iii) $\therefore$ from (i), $A=1-B$ ---(iv)

Put (iv) into (ii); $-4A-5B+C=0$ ---(ii)

$-4(1-B)-5B+C=0$, $-4+4B-5B+C=0$

$= -4-B+C=0$ ---(v) put (iv) into (iii)

$\therefore \overset{4(1-B)}{\cancel{4A}}+6B-3C=1 = 4-1+2B-3C=0$

$= 3+2B-3C=0$, from (v) form (vi) $\therefore -4-B+C=0$

$\therefore B = C - 4$ ---(vi) put (vi) into $3 + 2B - 3C = 0$

$3 + 2(C-4) - 3C = 0, \Rightarrow 3 + 2C - 8 - 3C = 0$

$\therefore -5 - C = 0 \therefore C = -5$

from (v), $-4 - B + C = 0$

$-4 - B - 5 = 0, \; -B - 9 = 0, \; \therefore B = 9$

Since $A + B = 1$, $\therefore A - 9 = 1 \quad \therefore A = 1 + 9 = 10$

$\therefore A = 10, B = -9$ and $C - 5$

$$\frac{x^2+1}{(x-3)(x+2)^2} = \frac{10}{(x-3)} - \frac{9}{(x-2)} - \frac{5}{(x-2)^2}$$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx - \int \frac{9}{(x-2)}\,dx - \int \frac{5}{(x-2)^2}\,dx$$

$$= 10\int \frac{dx}{(x-3} - 9\int \frac{dx}{(x-2)} - 5\int \frac{dx}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) + 5/(x-2) + C$$

ans.

d) $(x^3 + x^2 + x + 1)/(x-1)\,dx$

solution

$$\int \frac{x^3+x^2+x+1}{(x-1)}\,dx = \int \frac{x^3}{x-1}\,dx + \int \frac{x^2}{x-1}\,dx + \int \frac{x}{x-1}\,dx + \int \frac{dx}{x-1}$$

P.T.O

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using long division, consider the individual expressions

$\int \frac{x^3}{x-1}dx$, $\frac{x^3}{x-1}$ = (note the signs changes)

$$\begin{array}{r} x^2+x+1 \\ x-1 \overline{) x^3} \\ -x^3+x^2 \\ \hline x^2 \\ -x^2+x \\ \hline x \\ -x+1 \\ \hline 1 \end{array}$$

remainder

$\therefore \frac{x^3}{x-1} = x^2+x+1+\frac{1}{x-1}$

$\int \frac{x^3}{x-1} = \int x^2+x+1+\frac{1}{x-1}\,dx$

$\int \frac{x^3}{x-1} = \int x^2 dx + \int x\,dx + \int dx + \int \frac{1}{x-1}dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(x-1) + C$

$\int \frac{x^2}{x-1}dx$, $\frac{x^2}{x-1}$ =

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2} \\ -x^2+x \\ \hline x \\ -x+1 \\ \hline 1 \end{array}$$

remainder

$\therefore \frac{x^2}{x-1} = x+1+\frac{1}{x-1}$

$\int \frac{x^2}{x-1}dx = \int \left(x+1+\frac{1}{x-1}\right)dx$

$= \int x+1\,dx + \int \frac{dx}{x-1}$

$= \int x\,dx + \int dx + \ln(x-1)$

$= \frac{x^2}{2} + x + \ln(x-1) + C$

$* \int \frac{x}{x-1}\,dx, \quad \frac{x}{x-1} = x-1 \overline{)\ x}$

$-x+1$

1 remainder

$\therefore \frac{x}{x-1} = 1 + \frac{1}{x-1}, \quad \int \frac{x}{x-1}\,dx = \int dx + \int \frac{dx}{x-1}$

$= x + \ln(x-1) + C$

$* \int \frac{dx}{x-1} = \ln(x-1) + C$

Combining everything, we have

$\frac{x^3}{3} + \frac{x^2}{2} + x + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + x + \ln(x-1)$
$+ \ln(x-1)$, collect like terms

$= \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^2}{2} + 3x + 4\ln(x-1)$

$= \frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4\ln(x-1)$

$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$

ans.