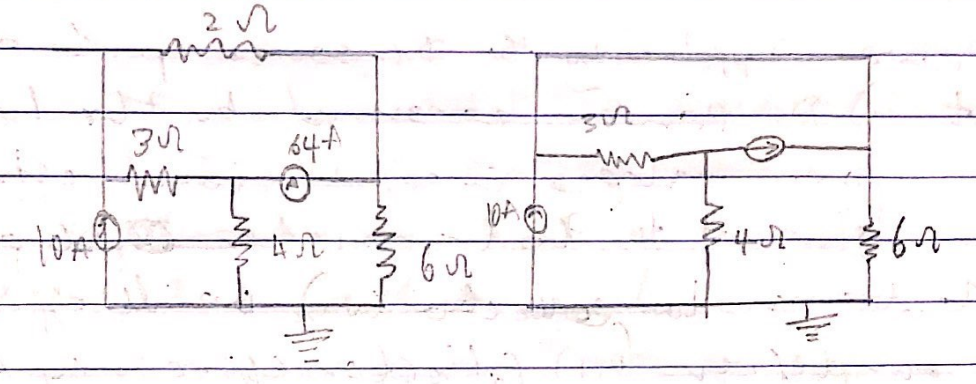
17/Eng02/1034
Computer Engineering
EEE 352

1) Find the voltages at nodes 1, 2 and 3. In the circuit below.



At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3}$$

$$\Rightarrow 60 = 3(v_1 - v_3) + 2(v_1 - v_2)$$

$$60 = 3v_1 - 3v_3 + 2v_1 - 2v_2$$

$$60 = 5v_1 - 2v_2 - 3v_3 \quad \cdots (i)$$

At Node 2, KCL:

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{v_1 - v_2}{3} - \frac{v_2 - 0}{4}$$

$$768 = 4(v_1 - v_2) - 3(v_2 - 0)$$

$$768 = 4v_1 - 4v_2 - 3v_2$$

$$768 = 4v_1 - 7v_3 \quad \cdots (ii)$$

At Node 3, KCL:

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{v_3 - 0}{6} - \frac{v_1 - v_3}{3}$$

$$384 = v_3 - 3(v_1 - v_3)$$

$384 = -3V_1 + 4V_3$ ---(iii)

Using Cramer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ --(i)

$4V_1 - 7V_2 = 768$ --(ii)

$-3V_1 + 4V_3 = 384$ --(iii)

In Matrix

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & 7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$; $V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$= 60(-28-0) - 768(-8-0) + 384(0-21)$

$= 1680 + 6144 - 8064$

$= -3600$

$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80\,V$

for V_2: $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5(768 \times 4 - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - (768 - 3))$

$= 2880\,\Omega$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64\,V$$

for V_3:

$$\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ 3 & 6 & 384 \end{vmatrix}$$

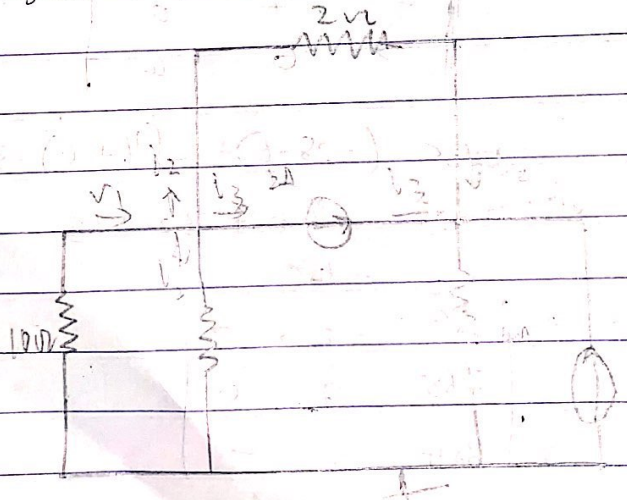
$$= 5((-7\times384)-6) - 4((-2\times384-6) - 3((-2\times768) - (-7\times60)))$$

$$= -7020$$

$$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156\,V$$

Hence $V_1 = 80\,V$, $V_2 = -64\,V$, $V_3 = 156\,V$

2) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below.



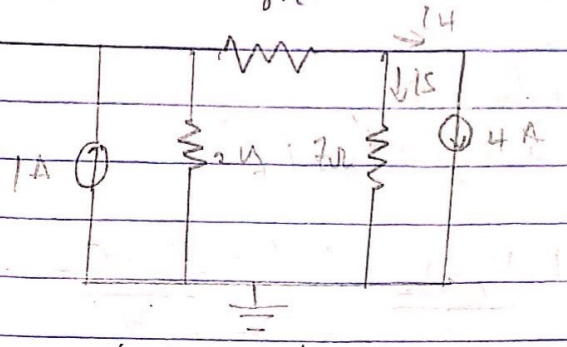
At Node 1, KCL

$$i_1 = i_3 + i_4$$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 10 + 2(V_1 - 0)$$

obtain V_1 and V_2 and the currents through the resistor in the circuit in example (ii) If the 2A current source and replaced by a 1A current source, or



At Node 1

$i_1 = i_2 + i_3$

$\frac{1}{1} = \frac{v_1 - v_2}{6} + \frac{v_1}{2}$

$6 = v_1 - v_2 + 3v_1$

$6 = 4v_1 - v_2$ --- (i)

At Node 2

$i_2 = i_4 + i_3$

$\frac{v_1 - v_2}{6} = 4 + \frac{v_2}{7}$

$7(v_1 - v_2) = 168 + 6v_2$

$168 = 7v_1 - 13v_2$ --- (ii)

From equation (i) $v_2 = 4v_1 - 6$

subs $(v_2 = 4v_1 - 6)$ in eqn(ii)

$168 = 7v_1 - 13(4v_1 - 6)$

$168 = 7v_1 - 52v_1 + 78$

$90 = -45v_1$

$v_1 = \frac{90}{-45}$

$v_1 = -2v$

Sub $v_1 = -2$ in eqn(i)

$6 = 4(-2) - v_2$

~~$-V_1 = 5V_1 - 5V_2 + 120 + 12V_1$~~

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ ---①

At Node 2

$i_3 + i_2 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2V(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ ---(ii)

Using Elimination Method

$120 = -8V_1 + 5V_2$ ---(i) $\times -4$

$144 = -4V_1 + 6V_2$ ---(ii) $\times -8$

$-480 = 32V_1 - 20V_2$ ---(iii)

$-1152 = 32V_1 - 48V_2$ ---(iv)

Substract eqn (iii) from eqn (iv)

$-672 = 0 - 28V_2$

$V_2 = \frac{-672}{-28}$

$V = 24$ V

Subs $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6V_2}{-4}$

$V_1 = 0$

$V_1 = 0$ V, $V_2 = 24$ V

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$\therefore V_1 = -2V, \quad V_2 = -14V$

Current through resistors

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{14}{7} = -2A$$