$C = 4 - 9$

$C \cancel{-4} = -9$

$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{10}{(x-3)} \frac{-9}{x-2} \frac{-5}{(x-2)}$

$\int \frac{10}{x-3} dx = \int \frac{-9}{x-2} dx \int \frac{-5}{(x-2)^2} dx$

$= 10 \ln(|x-3|) - 9\ln(|x-2|) + 5/x-2$

$10 \ln(|x-3|) - 9\ln(|x-2|) + \frac{5}{x-2} + c$

$\ln(|x-3|) = 10$

$\ln(|x-2|) = -9$

$\ln \frac{5}{(x-2)} = C$

4 $\frac{x^3+x^2x+1}{x-1} dx$

$\frac{x^3}{x-1} + \frac{x^2}{x-1} + \frac{x}{x-1} + \frac{1}{x-1} dx$

$\int \frac{x^3}{x-1} dx + \int \frac{x^2}{x-1} dx + \int \frac{x}{x-1} dx + \int \frac{1}{x-1} dx$

$\frac{2x^6+3x^2+(x-11}{6} + \ln(|x-1|) + \frac{x^2}{2} + x + \ln(|x-1|)$

$+ x - 1 + \ln(|x-1|) + \ln(|x-1|) + C$

$= \frac{2x^3+3x^2+6x}{6} - 11 + 4\ln(|x-1|) + C$

Name: [illegible]

Dept: [illegible]

Matric no: 19/SC[illegible]

Course Code: MAT 104

MAT 104 Assignment

1. $(3x-1)/(x-1)(x-2)(x-3)\,dx$
2. $(x^2+x+1)/(x+2)(x^2+1)\,dx$
3. $(x^2+1)/(x-3)(x-2)^2\,dx$
4. $x^3+x^2+x+1/(x^2-1)\,dx$

Solution

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$3x-1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$

$3x-1 = (A+B+C)x^2 + (-5A+4B-3C)x + (6A+3B+2C)$

Comparing | Equating Coefficients

$A+B+C = 0$ ---- (i)

$-5A+4B-3C = 3$ ---- (ii)

$6A+3B+2C = -1$ ---- (iii)

$A+B+C = 0$

$A = -B-C$ ---- (iv)

Put equation (iv) in (ii) & (iii)

$-5A-4B-3C = 3 \Rightarrow -5(-B-C)-4B-3C = 3$

$6A+3B+2C = -1 \quad 6(-B-C)+3B+2C = -1$

$B+2C = 3$ ---- (v) $\times -4$

$-3B-4C = -1$ ---- (vi) $\times 2$

$-6B-8C = +12$

$\underline{-2B-8C = -2}$

$B = -5$

Next page for continuation

[illegible] [illegible] (iii)

Put $B = -5$ in $C = 4$ in equation 1

$B + 2C = 3$

$-5 + 2C = 3$

$2C = 5 + 3$

$C = 4$

$A + B + C = 0$

$A - 5 + 4 = 0$

$A - 1 = 0$

$A = 1$

$$\int \frac{3x - 1}{(x-1)(x-2)(x-3)}\,dx \quad \int \frac{1}{(x-1)}\,dx + \int \frac{-5}{(x-2)}\,dx + \int \frac{4}{(x-3)}\,dx$$

$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + c$

$\ln(x-1) = -5$

$\ln(x-2) = +4$

$\ln(x-3) = +c$

2 $\dfrac{x^2 + x + 1\,dx}{(x+2)(x^2+1)} = \dfrac{A}{(x+2)} + \dfrac{(Bx + C)}{(x^2+1)}$

$\dfrac{x^2+x+1\,dx}{(x+2)(x^2+1)} \quad \dfrac{A(x^2+1) + (Bx + C)(x+2)}{(x+2)(x^2+1)}$

$x^2 + x + 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$x^2 + x + 1 = (A+B)x^2 + (2B + C)x + (A + 2C)$

Equating coefficients

$A + B = 1$ ---- (i)

$2B + C = 1$ ---- (ii)

$A + 2C = 1$ ---- (iii)

$A = 1 - B$ ---- (iv)

put equation (iv) into (ii)

(A, B, C) $\left(\frac{3}{5}\right)$ $\left(\frac{2}{5}\right)$ $\left(\frac{1}{5}\right)$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$= \frac{3/5}{x+2} + \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1}$$

$$= \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$= \int \frac{3}{5(x+2)}dx + \int \frac{2x+1}{5(x^2+1)}dx$$

$$= 3/5 - \ln(|x+2|) + 1/5 + \ln(x^2+1) + 2x$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1)\frac{2x+1}{5} + C$$

3. $\frac{x^2+1}{x-3)(x-2)^2}\ dx$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+1 = (x-2)^2 A(x-3) + (x-3)A + (x-3)$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$= Ax^2 + Bx^2 - 4Ax - 5Bx + Cx + 4A + 6B - 3C$$

$$x^2+1 (A+B)x^2 + (-4A-5B+C)x + (4A+6B-3C)$$

~~A=B~~ $A+B=1$ --- (i)

$-4A-5B+C=0$ --- (ii)

~~A=1~~

$4A+6B-3C=1$ --- (iii)

$A, B, C = (10, -9, -5)$

Put B in equation (i)

$A+B=1$

$A+(-9)=1$

$A=10$

Put b in equation (ii)

$-B+C=4$

$-(-9)+C=4$