MAT 104 ASSSIGNMENT

1. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

2. First,

Partial - fraction decomposition

$$\frac{3x-1}{(x-1)(x-2)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Multiplying both sides $(x-1)(x-2)(x-3)$

$$3x-1 \equiv (x-2)(x-3)A + B(x-1)(x-3) + C(x-1)(x-2)$$

$$3x-1 \equiv Ax^2 - 5A + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3C + 2C$$

$$3x-1 \equiv Ax^2 + Bx^2 + Cx^2 - 5Ax - 4Bx - 3Cx + 6A + 3B + 2C$$

$$3x^2 - 1 \equiv (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$$

$$-1 = 6A + 3B + 2C \quad \ldots (1)$$

$$3 = -5A - 4B - 3C \quad \ldots (2)$$

$$0 = A + B + C \quad \ldots (3)$$

From (3)

$$A = -B - C$$

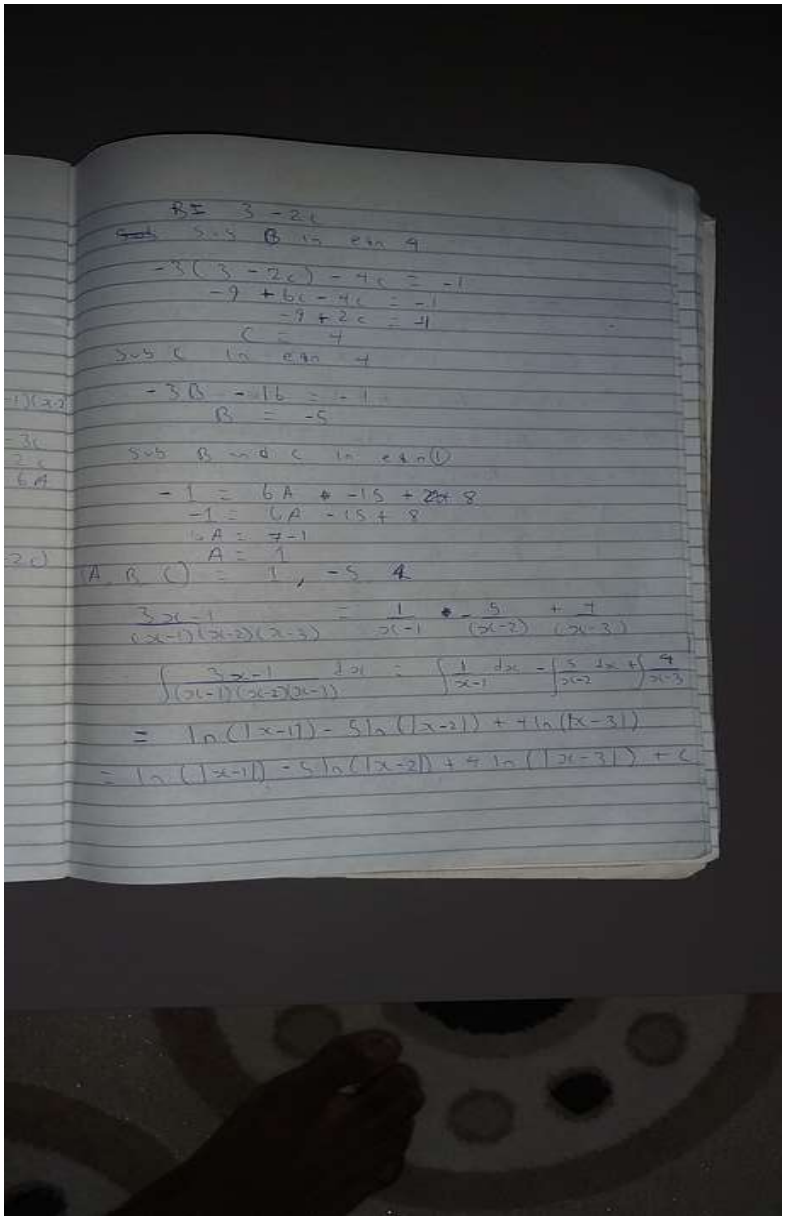
Sub in eqn (1) and (2)

$$6(-B-C) + 3B + 2C = -1$$

$$-5(-B-C) - 4B - 3C = 3,$$

$$-3B - 4C = -1 \quad \ldots (4)$$

$$B + 2C = 3 \quad \ldots (5)$$



B = 3 - 2c

Sub B in eqn 4

$-3(3-2c) - 4c = -1$

$-9 + 6c - 4c = -1$

$-9 + 2c = -1$

$c = 4$

Sub c in eqn 4

$-3B - 16 = -1$

$B = -5$

Sub B and c in eqn ①

$-1 = 6A + 15 + 8$

$-1 = 6A + 15 + 8$

$6A = -1$

$A = 1$

$A, B, C = 1, -5, 4$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-1)}\,dx = \int \frac{1}{x-1}\,dx - \left(\int \frac{5}{x-2}\,dx + \int \frac{4}{x-3}\right)$$

$= \ln(|x-1|) - 5\ln(|x-2|) + 4\ln(|x-3|)$

$= \ln(|x-1|) - 5\ln(|x-2|) + 4\ln(|x-3|) + C$