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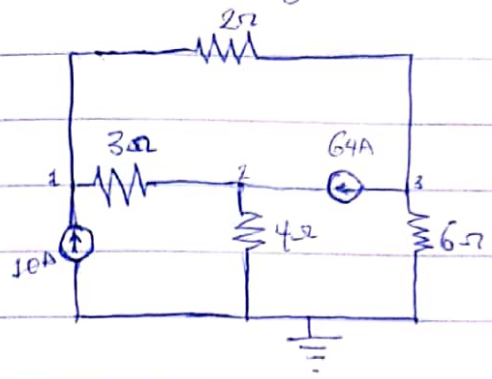
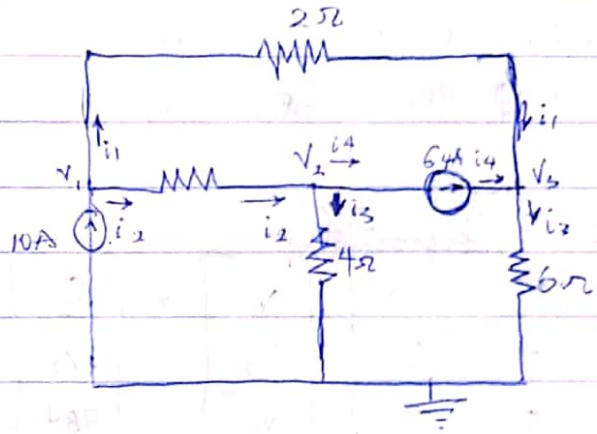
17/ENG04/038
Elect/Elect.
ENG 322 Assignment

1) Find the voltages at nodes 1, 2, 3 in the circuit below





At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \cdots (1)$$

At node 2, KCL

$$i_2 = i_3 + 64$$

$$i_2 - i_3 = 64$$

$$\frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4} = 64$$

$$4(V_1 - V_2) - 3(V_2 - 0) = 768$$

$$4V_1 - 4V_2 - 3V_2 = 768$$

$$4V_1 - 7V_2 = 768 \quad \cdots (11)$$

At node 3, KCL

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \quad \ldots (iii)$

Using Cramers Rule

$5V_1 - 2V_2 - 3V_3 = 60 \quad \ldots (i)$

$4V_1 - 7V_2 - 0 = 768 \quad \ldots (ii)$

$-3V_1 + 0 + 4V_3 = 384 \quad \ldots (iii)$

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$; $V_2 = \frac{\Delta_2}{\Delta}$; $V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ 3 & 0 & 4 \end{vmatrix}$

$= 5(-28+0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$= 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For V_2; $\Delta_2 =$

$$\begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$= 5[(768 \times 4) - 0] - 4[(60 \times 4) - (384 - 3)] - 3[0 - (768 \times -3)]$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

For V_3;

$$\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 768) - (-7 \times 60)]$

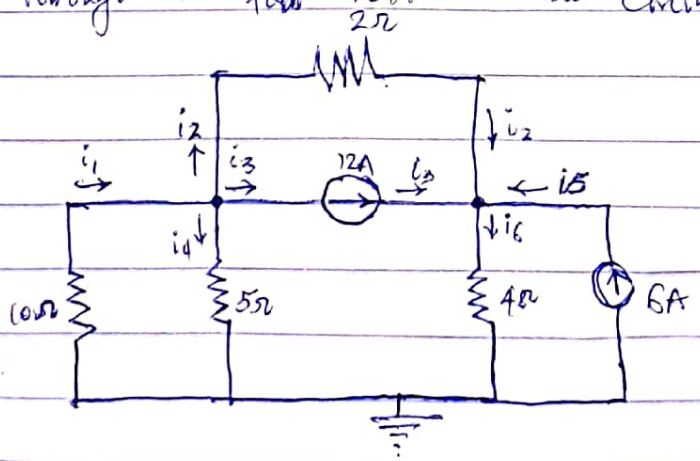
$= -7020$

$\therefore \frac{V_3 = \Delta_3}{\Delta} = \frac{-7020}{-45}$

$= 156V$

Hence $V_1 = 80V$; $V_2 = -64V$, $V_3 = 156V$

2) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below.



At Node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \quad \text{---} (i)$$

At Node 2 ; KCL

$$i_8 + i_4 + i_5 = i_6$$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2V_2$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \quad \text{---} (ii)$$

Using Elimination method ;

$$120 = -8V_1 + 5V_2 \quad \text{---} (i) \quad \times -4$$

$$144 = -4V_1 + 6V_2 \quad \text{---} (ii) \quad \times -8$$

$$-480 = 32V_1 - 20V_2 \quad \text{---} (iii)$$

$$-1152 = 32V_1 - 48V_2 \quad \text{---} (iv)$$

Subtract (iii) from (iv)

$$-672 = 0 - 28V_2$$

$$V_2 = \frac{-672}{-28}$$

$$V_2 = \frac{24V}{2}$$

put $V_2 = 24V$ in eqn (ii)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6V_2}{4}$$

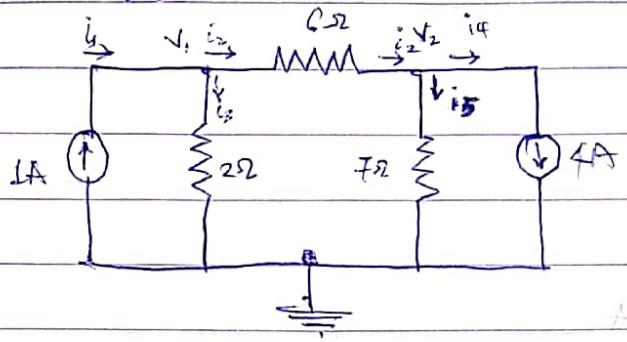
$$V_1 = \frac{144 - 6(24)}{4}$$

$$V_1 = \frac{0}{4} = 0$$

$V_1 = 0V$; $V_2 = 24V$

$i_1 = 0A$; $i_2 = 0A$; $i_3 = 6A$; $i_4 = -12A$

ii) Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (a) if the 2A current source was replaced by a 1A current source.



At node 1 ; KCL

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ ---- (i)

At node 2 ; KCL

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ ---- (ii)

From eqn (i), $V_2 = 4V_1 - 6$

Put $V_2 = 4V_1 - 6$ in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = \frac{90}{-45} = -2V$

Put $V_1 = -2$ in eqn (1)

$$6 = 4(-2) - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = -14V$$

$\therefore V_1 = -2V$, $V_2 = -14V$

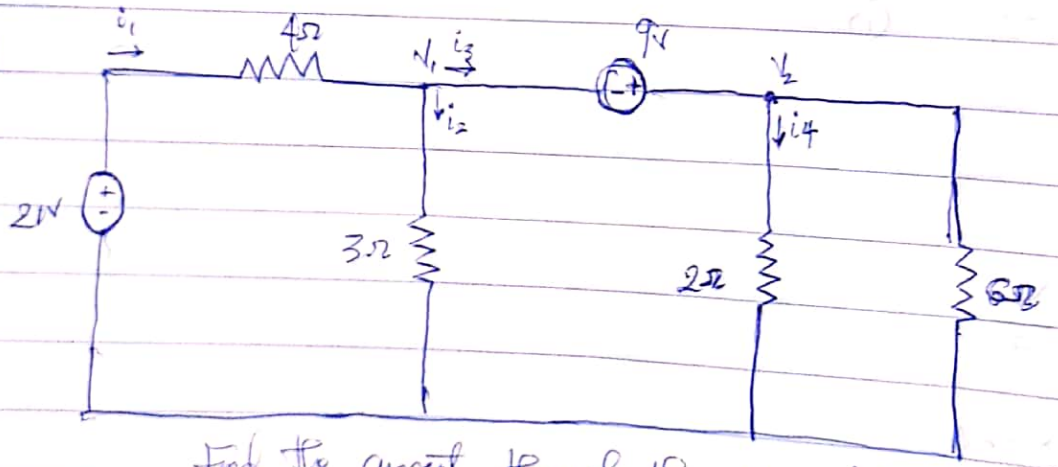
Current through the resistors:

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_4 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$

3)



Find the current through the 3Ω and 2Ω resistors.

Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 21}{4} + \frac{V_1}{2} + \frac{V_2}{6} + \frac{V_2}{2} = 0$$

$$7V_1 + 8V_2 - 63 = 0 \quad -(1)$$

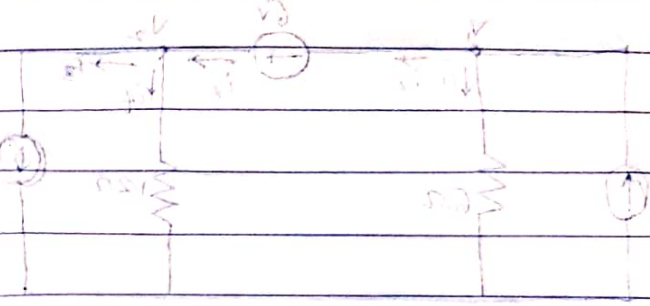
Using KVL for Loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9 \quad \ldots (ii)$

$7V_1 + 8V_2 = 63 \quad — (iii)$

$-V_1 + V_2 = 9 \quad — (iv)$



from (iv) Let $V_2 = 9 + V_1$

Put V_2 in eqtn (iii)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V_1 = -0.6V$

Put $V_1 = 0.6V$ in eqtn (iv)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 9 - 0.6$

$V_2 = 8.4V$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

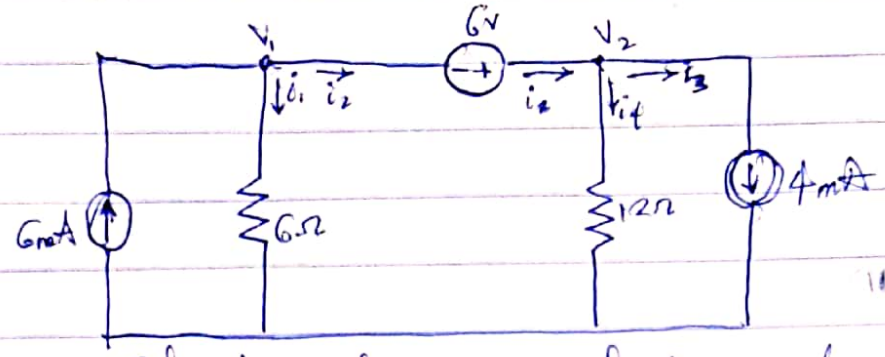
Current through the 3Ω resistor;

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4)



Find the node voltages and the currents through the 6Ω and 12Ω resistors.

Soln

Assuming that $V_1 - V_2 = 6V \Rightarrow i_2$

node 1 ; Using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$36 = V_1 + 6(V_1 - V_2)$$

$$36 = V_1 + 6V_1 - 6V_2$$

$$36 = 7V_1 - 6V_2 \quad \cdots (1)$$

node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 48 + V_2$$

$$48 = 12V_1 - 12V_2 - V_2$$

$$48 = 12V_1 - 13V_2 \quad \text{— (ii)}$$

Using Elimination method.

$$36 = 7V_1 - 6V_2 \quad \text{— } \times 12$$

$$48 = 12V_1 - 13V_2 \quad \text{— } \times 7$$

$$432 = 84V_1 - 72V_2$$

$$-\ 336 = 84V_1 - 91V_2$$

$$96 = 19V_2$$

$$V_2 = \frac{96}{19}$$

$$V_2 = 5.1V$$

Putting $V_2 = 5.1V$ in eqtn (i)

$$36 = 7V_1 - 6(5.1)$$

$$36 = 7V_1 - 30.6$$

$$7V_1 = 66.6$$

$$V_1 = \frac{66.6}{7}$$

$$V_1 = 9.5V$$

$\therefore V_1 = 9.5V$ and $V_2 = 5.1V$

$\therefore$ Current through the 6Ω resistor

$$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$$

Current through the 12Ω resistor

$$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$$

$\therefore V_1 = 9.5V$; $V_2 = 5.1V$

$i_1 = 1.58A$; $i_4 = 0.43A$