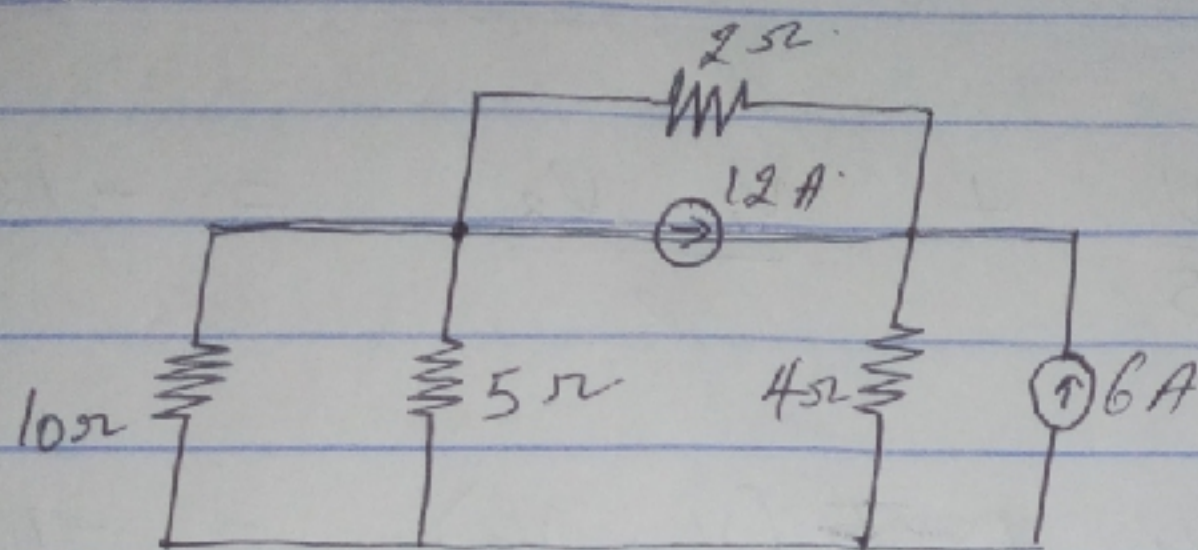


Iwatt Enobong 2160
 18/Eng 04/080
 Electrical / Electronics Engineering
 EEE 322 : Circuit Theory II

Assignment

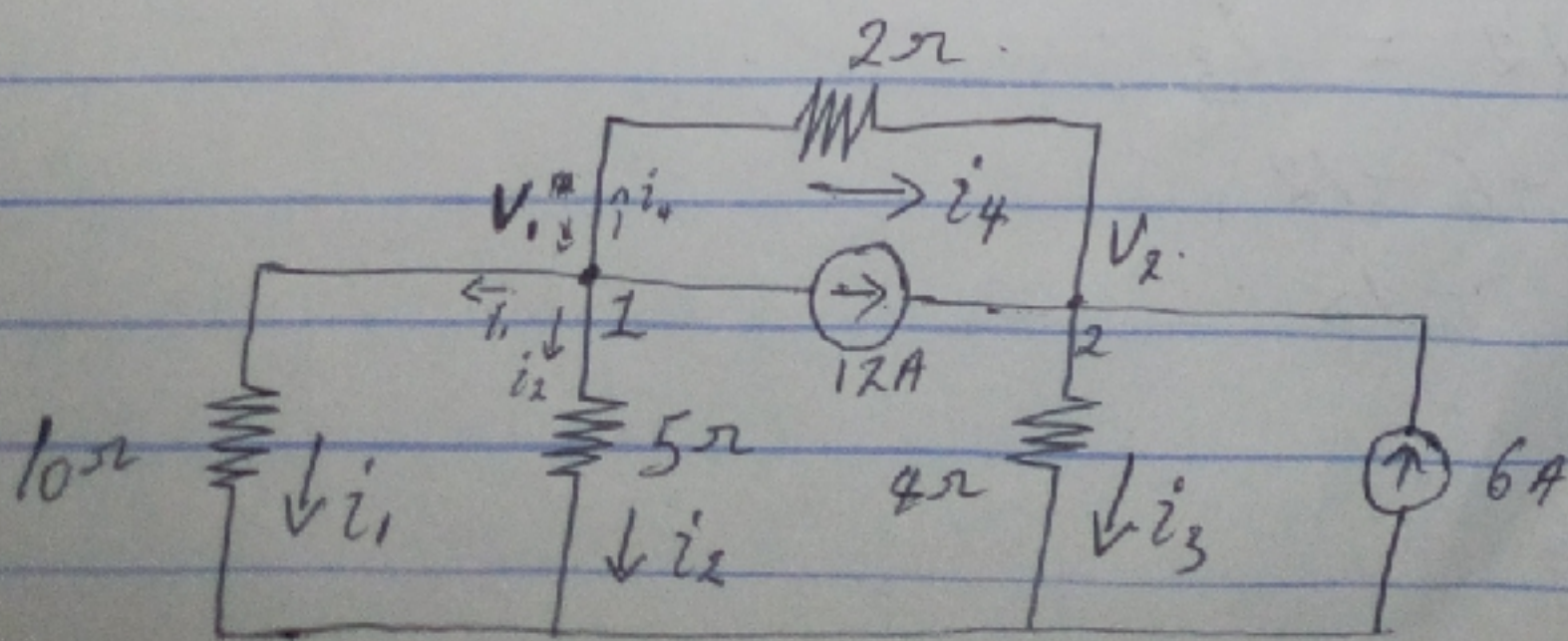
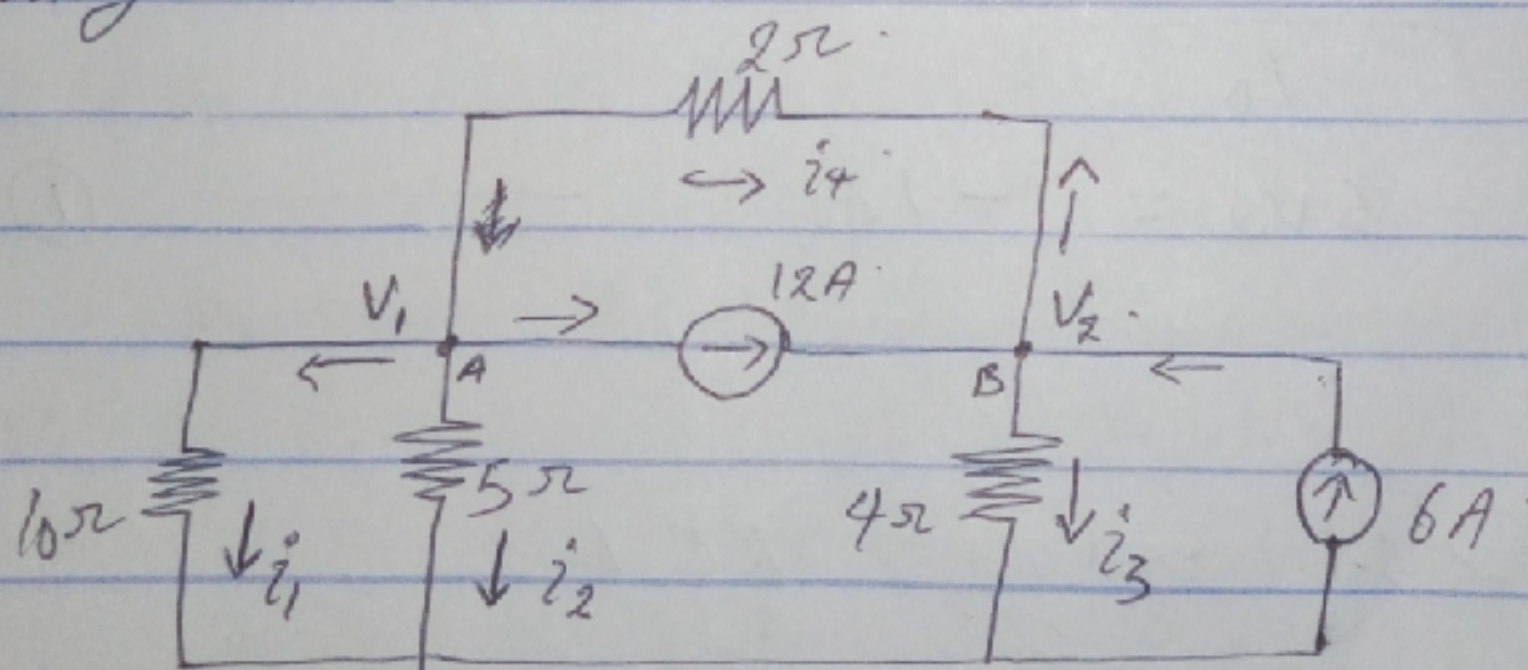
①



- i) Find the voltages at nodes 1 & 2
 ii) Find the current flowing through the four resistors.

Soln...

redrawing



②

from node 1; using KCL

$$(i_1 + i_2 + i_4 + 12 = 0)$$

$$i_1 + i_2 + i_4 = -12 A$$

$$\rightarrow i_1 = \frac{V_1 - 0}{10} = \frac{V_1}{10}$$

$$i_2 = \frac{V_1 - 0}{5} = \frac{V_1}{5}$$

$$i_4 = \frac{V_1 - V_2}{2}$$

$$\Rightarrow \frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -12$$

$$\equiv \frac{V_1}{10} + \frac{2V_1}{10} + \frac{5(V_1 - V_2)}{10} = -12$$

$$\Rightarrow \frac{V_1 + 2V_1 + 5V_1 - 5V_2}{10} = -12$$

$$\Rightarrow \frac{8V_1 - 5V_2}{10} = -12$$

$$\Rightarrow 8V_1 - 5V_2 = -120 \quad \text{--- (1)}$$

from node 2; using KCL

$$i_4 + 6 + 12 = i_3$$

$$i_4 - i_3 = -6 - 12$$

$$i_4 - i_3 = -18$$

$$i_4 = \frac{V_1 - V_2}{2}$$

3

3

$$i_3 = \frac{V_2 - 0}{4} = \frac{V_2}{4}$$

$$\Rightarrow \frac{V_1 - V_2}{2} - \frac{V_2}{4} = -18$$

$$\equiv \frac{2(V_1 - V_2)}{4} - \frac{V_2}{4} = -18$$

$$\Rightarrow \frac{2V_1 - 2V_2 - V_2}{4} = -18$$

$$\Rightarrow \frac{2V_1 - 3V_2}{4} = -18$$

$$\Rightarrow 2V_1 - 3V_2 = -72 \quad \text{--- (2)}$$

$$\begin{aligned} * \quad 8V_1 - 5V_2 &= -120 \\ 2V_1 - 3V_2 &= -72 \end{aligned}$$

using calculator to solve simultaneously, we get;
 $V_1 \equiv 0V$ and $V_2 \equiv 24V$

$$\therefore, V_1 = 0V \text{ and } V_2 = 24V //$$

$$\therefore i_1 = \frac{V_1}{10} = \frac{0}{10} = 0A$$

(4)

$$i_2 = \frac{V_1}{5} = \frac{0}{5}$$

$$= 0A$$

$$i_3 = \frac{V_2}{4} = \frac{24}{4}$$

$$= 6A$$

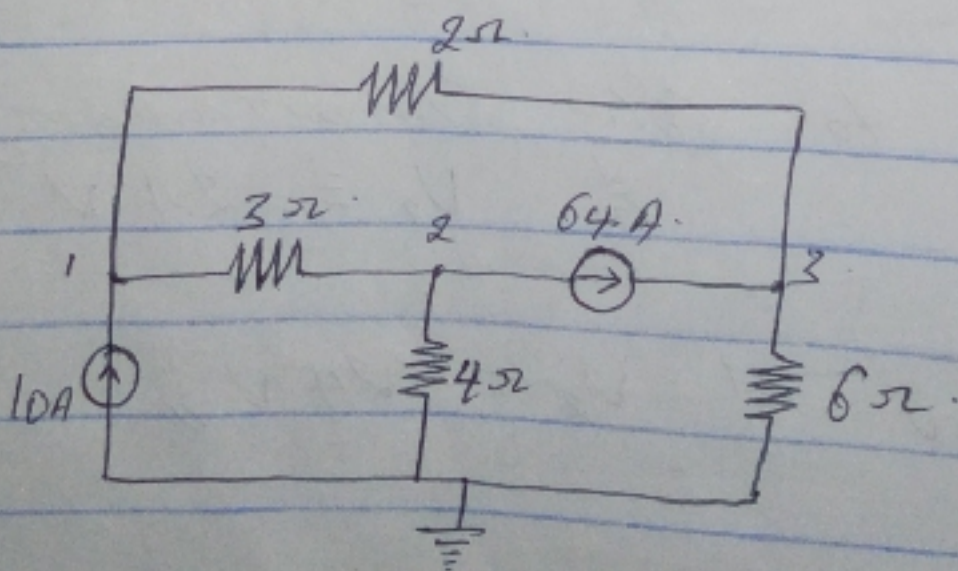
$$i_4 = \frac{V_1 - V_2}{2} = \frac{0 - 24}{2}$$

$$= -12A$$

$$\therefore; V_1 = 0V, \quad V_2 = 24V$$

$$i_1 = 0A, \quad i_2 = 0A, \quad i_3 = 6A \quad \& \quad i_4 = -12A$$

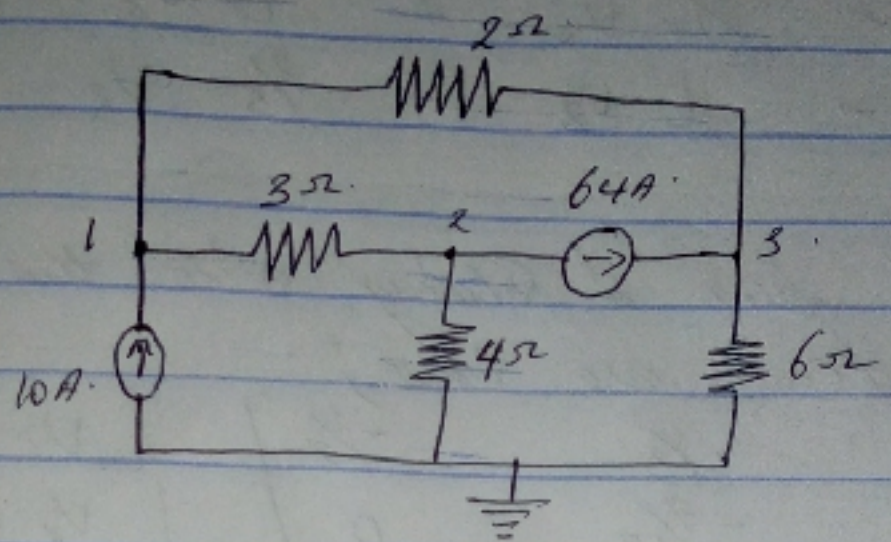
(2)



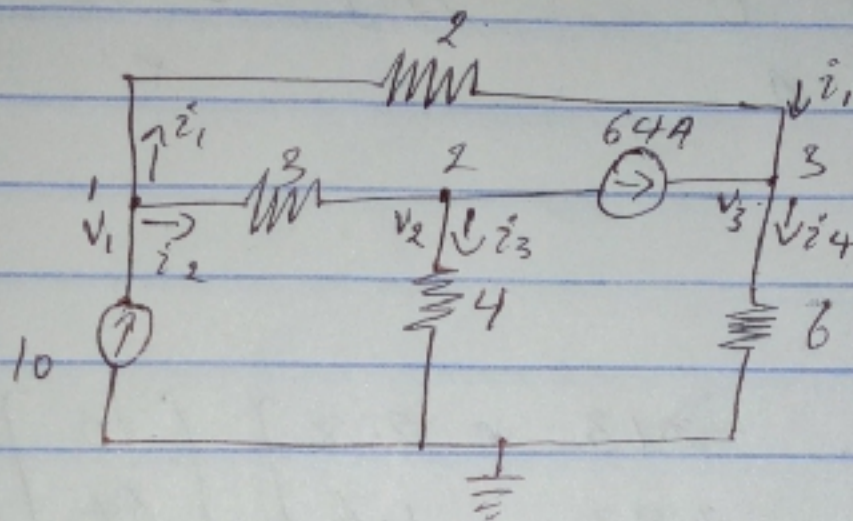
find the voltages at nodes 1, 2 & 3.

5

2



Soln



@ node 1; using KCL $\therefore 10 = i_1 + i_2$

$$10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

@ node 2; using KCL $\therefore i_2 = 64 + i_3$

$$\frac{V_1 - V_2}{3} = 64 + \frac{V_2}{4}$$

@ node 3; using KCL $\therefore i_4 = 64 + i_1$

$$\frac{V_3}{6} = 64 + \frac{V_1 - V_3}{2}$$

Simplifying the equations, we'll have;

(6)

$$10 = \frac{5}{6} V_1 - \frac{1}{3} V_2 - \frac{1}{2} V_3$$

$$64 = \frac{1}{3} V_1 - \frac{7}{12} V_2 + 0 V_3$$

$$64 = -\frac{1}{2} V_1 - 0 V_2 + \frac{2}{3} V_3$$

using matrix form & solving with the aid of MatCad, we have;

$$\begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix} = \begin{bmatrix} \frac{5}{6} & -\frac{1}{3} & -\frac{1}{2} \\ \frac{1}{3} & -\frac{7}{12} & 0 \\ -\frac{1}{2} & 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$b = \quad \quad \quad A \quad \quad \quad x$

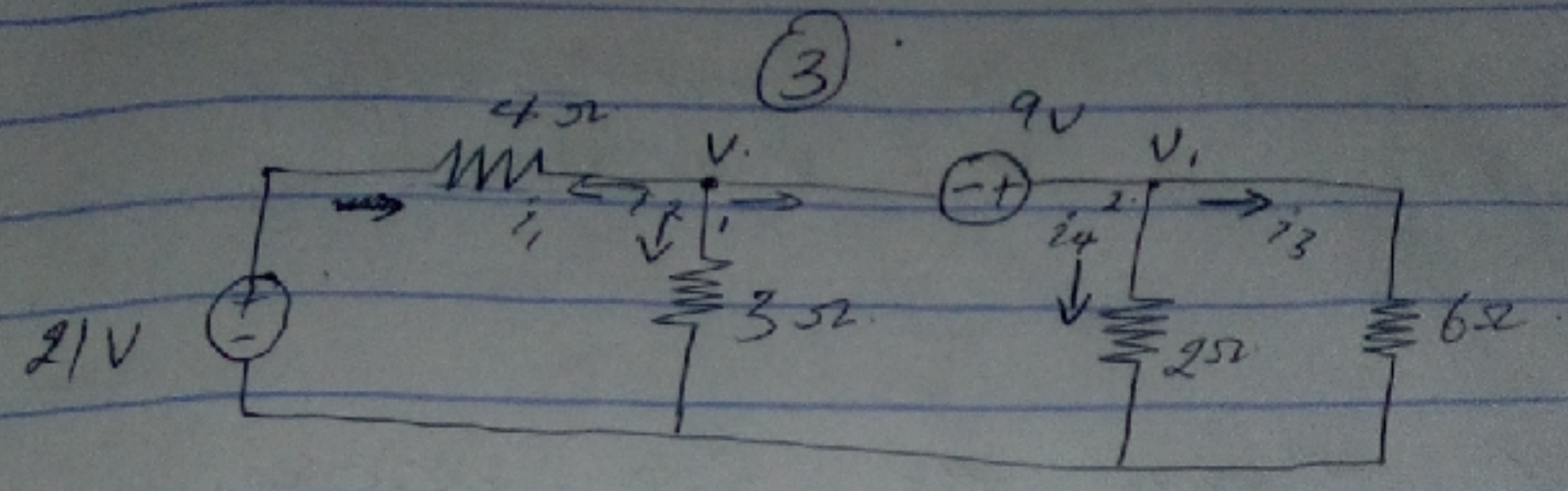
$$x = A^{-1} b$$

$$A^{-1} b = \begin{bmatrix} 3.73 & -2.13 & 2.8 \\ 2.13 & -2.93 & 1.6 \\ 2.8 & -1.6 & 3.6 \end{bmatrix} \begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix}$$

$$x = \begin{bmatrix} 80 \\ -64 \\ 156 \end{bmatrix}$$

$$\therefore V_1 = 80V, \quad V_2 = -64V \quad \& \quad V_3 = 156V$$

9



$$21 \parallel 6 = \frac{2 \times 6}{2+6} = 1.5$$

Find the current through the 3 & 2 resistor.

Soln...

Using KCL @ node 1.
 $i_1 + i_2 + i_3 + i_4 = 0$

$$\rightarrow \frac{V-21}{4} + \frac{V}{3} + \frac{V_1}{6} + \frac{V_1}{2} = 0$$

$$\Rightarrow \frac{3(V-21) + 4V + 2V_1 + 6V_1}{12} = 0$$

$$\rightarrow \frac{3V - 63 + 4V + 2V_1 + 6V_1}{12} = 0$$

$$\rightarrow 7V + 8V_1 - 63 = 0$$

$$7V + 8V_1 = 63 \quad \text{--- (1)}$$

Using KVL for loop 1.

$$\rightarrow V - 9 + V_1 = 0$$

$$-V + V_1 = 9 \quad \text{--- (2)}$$

$$7V + 8V_1 = 63$$

$$-V + V_1 = 9$$

⑧

using calculator to solve simultaneously,
 $V = -0.60$, $V_1 = 8.4 \text{ V}$

* the current through the 3Ω resistor;
using nodal analysis.

$$I_{3\Omega} = \frac{V - 0}{3}$$

$$= \frac{-0.6 - 0}{3} = \frac{-0.6}{3}$$

$$\therefore I_{3\Omega} = -0.2 \text{ A} \quad \dots \quad \text{ans 1}$$

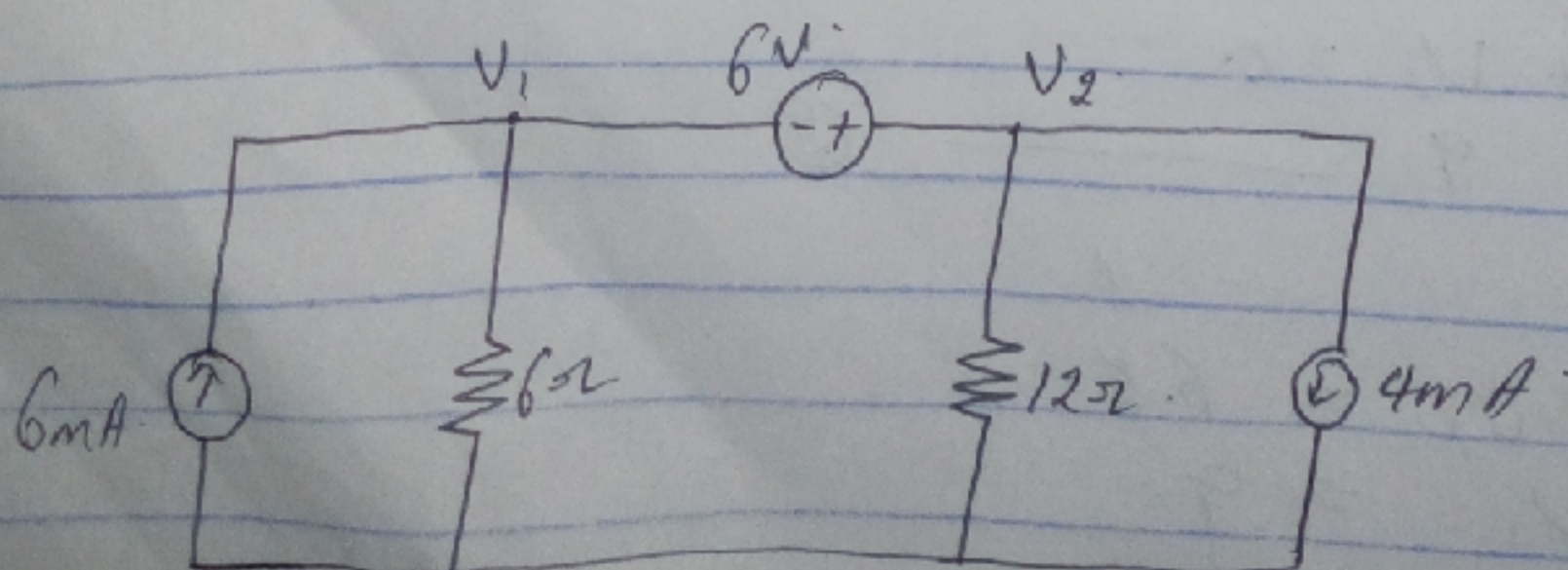
* the current through the 2Ω resistor;

$$I_{2\Omega} = \frac{V_1 - 0}{2}$$

$$= \frac{8.4 - 0}{2} = \frac{8.4}{2}$$

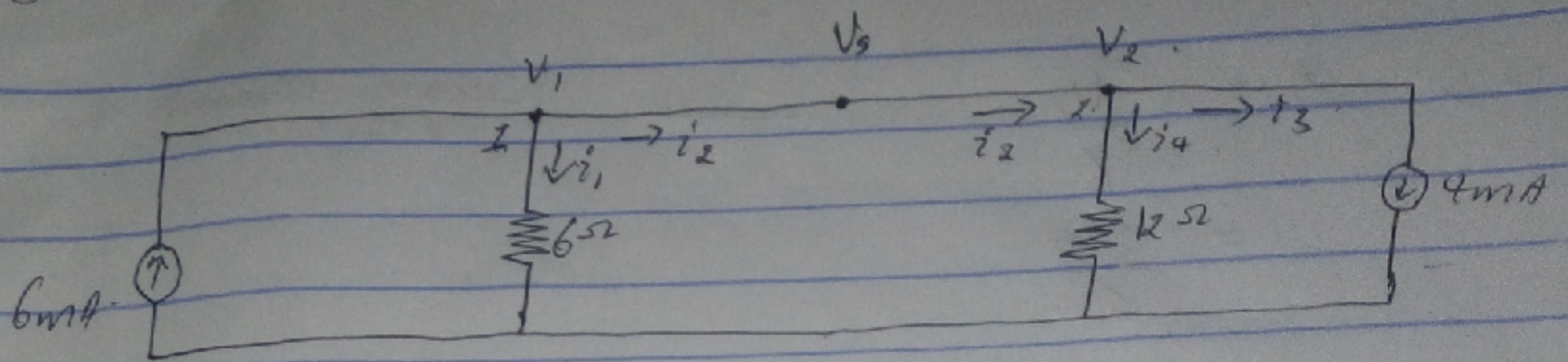
$$I_{2\Omega} = 4.2 \text{ A} \quad \dots \quad \text{ans 2}$$

no ④



find the voltages & currents through 6Ω & 12Ω .

9



* $\rightarrow V_1 - V_2 = 6V$

Let's say $V_1 - V_2 \equiv i_2$

@ node 1; using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1}{6} + (V_1 - V_2)$$

$$6mA = \frac{V_1 + 6(V_1 - V_2)}{6}$$

$$6mA = \frac{V_1 + 6V_1 - 6V_2}{6}$$

$$36 = 7V_1 - 6V_2 \quad \text{--- (1)}$$

@ node 2; using KCL

$$i_2 = i_4 + i_3$$

* $i_3 = 4mA$

$$\rightarrow i_2 = i_4 + 4mA \quad \therefore 4mA = i_2 - i_4$$

$$4mA = (V_1 - V_2) - \frac{V_2}{12}$$

$$4mA = \frac{12(V_1 - V_2) - V_2}{12}$$

$$48 = 12V_1 - 12V_2 - V_2$$

$$48 = 12V_1 - 13V_2 \quad \text{--- (2)}$$

Solving V_1 & V_2 simultaneously, we have.

$$V_1 = 9.5V, \quad V_2 = 5.1V$$

Current through
6Ω

$$\begin{aligned} \therefore i_1 &= \frac{V_1}{6} \\ &= \frac{9.5}{6} \end{aligned}$$

$$= 1.58 \text{ A}$$

$$\begin{aligned} i_2 &= V_1 - V_2 \\ &= 9.5 - 5.1 \\ &= 4.4 \text{ A} \end{aligned}$$

Current through
the 12Ω.

$$\begin{aligned} i_{12} &= \frac{V_2}{12} \\ &= \frac{5.1}{12} \end{aligned}$$

$$= \cancel{0.425} \text{ A} = 0.43 \text{ A}$$

$$\therefore V_1 = 9.5 \text{ V}, \quad V_2 = 5.1 \text{ V}$$

$$\therefore i_{6\Omega} = 1.58 \quad \therefore i_{12\Omega} = 0.43 \text{ A}$$