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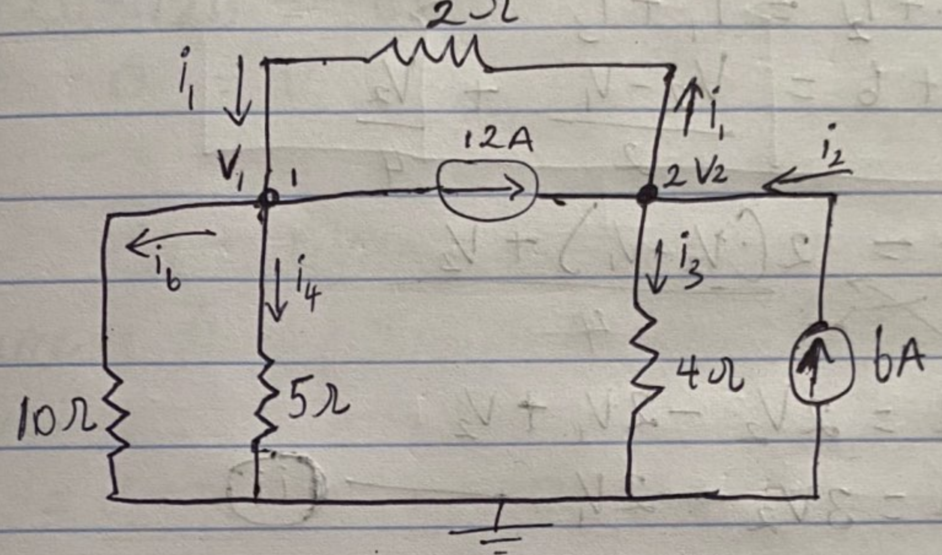
ELECTRICAL AND ELECTRONICS ENGINEERING

Circuit Theory II Assignment FTT

Question 1



Soln

Applying KCL at node 1

$i_1 = i_6 + i_4 + 12$

$$\frac{V_2 - V_1}{2} = \frac{V_1}{10} + \frac{V_1}{5} + \frac{12}{1}$$

$$\frac{V_2 - V_1}{2} = \frac{V_1 + 2V_1 + 120}{10}$$

$$\frac{5(V_2 - V_1)}{10} = \frac{V_1 + 2V_1 + 120}{10}$$

$$5V_2 - 5V_1 = V_1 + 2V_1 + 120$$

$5V_2 - 5V_1 - V_1 - 2V_1 = 120$

$5V_2 - 8V_1 = 120$ —— (i)

Applying KCL at node 2

$12 + i_2 = i_1 + i_3$

$$12 + 6 = \frac{V_2 - V_1}{2} + \frac{V_2}{4}$$

$$\frac{18}{1} = \frac{2(V_2 - V_1) + V_2}{4}$$

$72 = 2V_2 - 2V_1 + V_2$

$72 = 3V_2 - 2V_1$ —— (ii)

recalling:

$120 = -8V_1 + 5V_2$ —— (i)

$72 = -2V_1 + 3V_2$ —— (ii)

Applying elimination method

=> multiplying eqn (ii) by 4

$120 = \cancel{-8V_1} + 5V_2$

$288 = \cancel{-8V_1} + 12V_2$

$$\frac{-168}{-7} = \frac{\cancel{-7}V_2}{\cancel{-7}}$$

$V_2 = 24 \text{ Volts}$

$\therefore$ Substituting the Values of $V_2 = 24$ Volt into eqn (1)

$$120 = -8V_1 + 5(24)$$

$$120 - 120 = -8V_1$$

$$\frac{0}{-8} = V_1$$

$$V_1 = 0 \text{ Volts}$$

$\Rightarrow$ The Currents flowing Through Resistors: $6\Omega, 2\Omega, 5\Omega$ & 4Ω are:

$$i_1 = \frac{V_2 - V_1}{2} = \frac{24 - 0}{2} = 12A$$

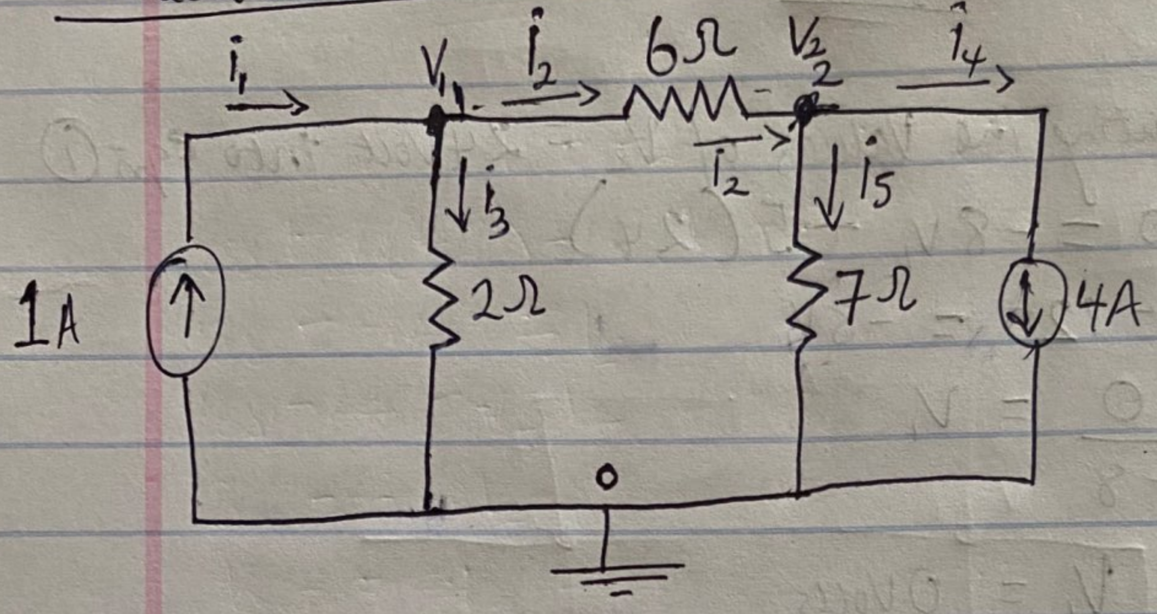
$$i_6 = \frac{V_1}{10} = \frac{0}{6} = 0A$$

$$i_4 = \frac{V_1}{5} = \frac{0}{5} = 0A$$

$$i_3 = \frac{V_2}{4} = \frac{24}{4} = 6A$$

Scanned with CamScanner

Question 2



Soln

Applying KCL at node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$\frac{1}{1} = \frac{V_1 - V_2 + 3V_1}{6}$

$6 = 4V_1 - V_2$

Applying KCL at node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = \frac{4}{1} + \frac{V_2}{7}$

Scanned with CamScanner

$$\frac{V_1 - V_2}{6} = \frac{28 + V_2}{7}$$

$$\frac{7(V_1 - V_2) = 6(28 + V_2)}{42}$$

$$\frac{(7V_1 - 7V_2 = 168 + 6V_2)}{\cancel{42}} \times \cancel{42}$$ (multiply Through by 42)

$7V_1 - 7V_2 = 168 + 6V_2$

$7V_1 - 7V_2 - 6V_2 = 168$

$7V_1 - 13V_2 = 168$ —— ②

recalling

$6 = 4V_1 - V_2$ —— ①

$168 = 7V_1 - 13V_2$ —— ②

from eqn ①

$-V_2 = 6 - 4V_1$

$\therefore V_2 \; -6 + 4V_1$ —— (*)

Putting eqn (*) into eqn ②

$168 = 7V_1 - 13(-6 + 4V_1)$

$168 = 7V_1 + 78 - 52V_1$

$168 - 78 = -45V_1$

$90 = -45V_1$

$V_1 = -2$ Volt

Substituting $V_1 = -2$ into eqn ①

$168 = 7(-2) - 13(V_2)$

$168 = -14 - 13V_2$

$168 + 14 = -13V_2$

$182 = -13V_2$

$V_2 = -14$ Volts

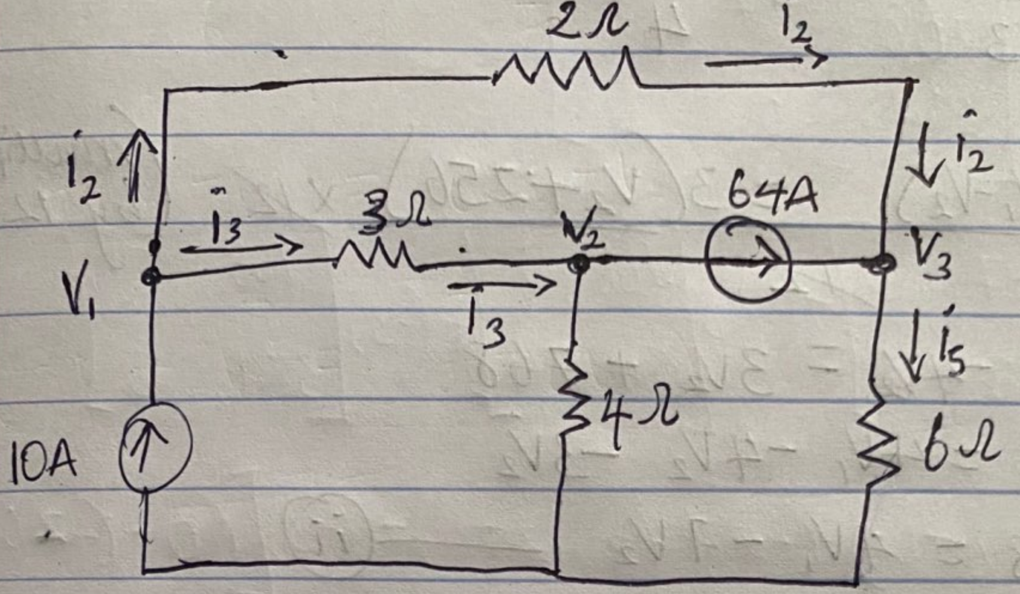
=> The currents flowing through resistors: 6Ω, 7Ω & 2Ω includes

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_2 = \frac{V_1 - V_2}{6} = 2A$

$i_5 = \frac{V_2}{7} = -2A$

Question 3



Soln

Applying KCL at node 1:

$$i_1 = i_2 + i_3$$

$$10A = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$10 = \frac{3(V_1 - V_3) + 2(V_1 - V_2)}{6}$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{—— (i)}$$

Applying KCL at node 2

$$i_3 = i_4 + 64A$$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{4} + \frac{64}{1}$$

$$\frac{V_1 - V_2}{3} = \frac{V_2 - 256}{4}$$

$$\frac{4(V_1 - V_2) = 3(V_2 + 256)}{\cancel{12}} \times \cancel{12}$$ (multiplying Through by 12)

$4V_1 - 4V_2 = 3V_2 + 768$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2$ —— (ii)

Applying KCL at node (3)

$64 + i_2 = i_5$

$$\frac{64}{1} + \frac{V_1 - V_3}{2} = \frac{V_3}{6}$$

$$\frac{128 + V_1 - V_3}{2} = \frac{V_3}{6}$$

$$\frac{3(128 + V_1 - V_3) = V_3}{\cancel{6}} \times \cancel{6}$$

$384 + 3V_1 - 3V_3 = V_3$

$384 = V_3 + 3V_3 - 3V_1$

$384 = 4V_3 - 3V_1$ —— (iii)

recalling:

$60 = 5V_1 - 2V_2 - 3V_3$ — (i)

$768 = 4V_1 - 7V_2$ — (ii)

$384 = -3V_1 + 4V_3$ — (iii)

Expressing the above in matrix form

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

recall: $V_1 = \frac{\Delta_1}{\Delta}$; $V_2 = \frac{\Delta_2}{\Delta}$, $V_3 = \frac{\Delta_3}{\Delta}$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

$\Delta = 5(-28-0) - (-2)(16-0) + (-3)(0-21)$

$\Delta = -140 + 32 + 63$

$\Delta = -45$

$$\Delta_1 = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix} \end{array}$$

$$\Delta_1 = 60\begin{bmatrix} -7 & 0 \\ 0 & 4 \end{bmatrix} - (-2)\begin{bmatrix} 768 & 0 \\ 384 & 4 \end{bmatrix} + (-3)\begin{bmatrix} 768 & -7 \\ 384 & 0 \end{bmatrix}$$

$$\Delta_1 = 60(-28-0) - (-2)(3072-0) + (-3)[(0-(-2688))]$$

$$\Delta_1 = -1680 + 6144 - 8064$$

$$\Delta_1 = -3600$$

Hence; $V_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{-3600}{-45} = 80$ Volts

Recalling eqn (iii), substitute $V_1 = 80$ volts into eqn (iii)

$$384 = -3(80) + 4V_3$$

$$384 = -240 + 4V_3$$

$$384 + 240 = 4V_3$$

$$624 = 4V_3$$

$$V_3 = 156 \text{ Volts}$$

Substituting the Values of V_1 & V_3 into eqn (i)

$60 = [80]5 - 2(V_2) - 3(156)$

$60 = 400 - 2V_2 - 468$

$60 - 400 + 468 = -2V_2$

$\frac{128}{-2} = \frac{\cancel{-2} V_2}{\cancel{-2}}$

$V_2 = -64$ Volts