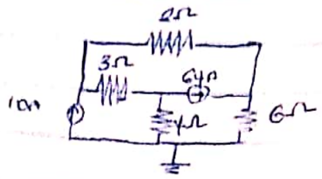
AHOH TUCHUKWU
17/ENG04/014
ELECTRICAL / ELECTRONICS ENGINEERING
EEE 322 Assignment

1) Find the Voltages at nodes 1, 2 and 3 in the Circuit below.



At node 1 KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{— (i)}$

at node 2, KCL:

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2 \quad \cdots \text{(ii)}$

at node 3, KCL:

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \quad \cdots \text{(iii)}$

Scanned with CamScanner

Using Crammer's Rule

5V - 2V2 - 3V3 = 60 ... (i)

4V1 - 7V2 = 768 ... (ii)

-3V1 + 4V3 = 384 ... (iii)

In matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$, $V_3 = \frac{\Delta_3}{\Delta}$

Where $\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$

$= 5(-28) + 12(16+0) - 3(0-21)$

$= 140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix} = 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 - 8064 = -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For $\Delta_2 = \begin{bmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{bmatrix} = 5(768 \times 4) - 0) - 4(60 \times 4) - (384 - 3) - 3(0 - (768 - 3))$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

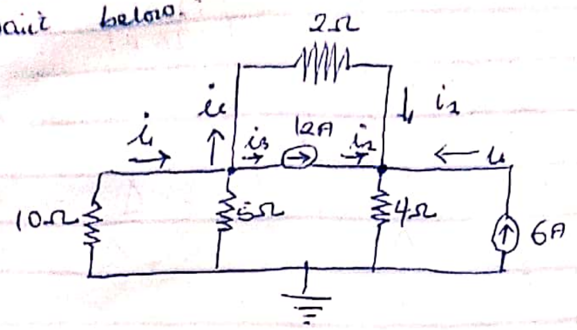
For V_3, $\Delta_3 = \begin{bmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ 3 & 0 & 384 \end{bmatrix}$

$= 5((-7 \times 384) - 0) - 4(-2 \times 384 - 0) - 3(-2 \times 768) -$

$(-7 \times 60) = -7020$

$V_3 = \frac{\Delta_4}{\Delta} = \frac{-7020}{-45} = 150\ V$

Q) Find the Voltages at node 1 and 2 and determine the Currents flowing through the four resistors in the circuit below.



At node 1, KCL.

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2V_1$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ —— (1)

At node 2

$i_3 + i_2 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$

Using Elimination Method

$120 = -8V_1 + 5V_2$ —— (1) $\times -4$

$144 = -4V_1 + 6V_2$ = (2) $\times -8$

$480 = 32V_1 - 20V_2$ —— (111)

$-1152 = 32V_1 - 48V_2$ —— (IV)

Subtract eqn (111) from eqn (IV)

$V_2 = \frac{-672}{-28}$

$V_2 = 24\,V$

subs $V_2 = 24$ in eqn (i)

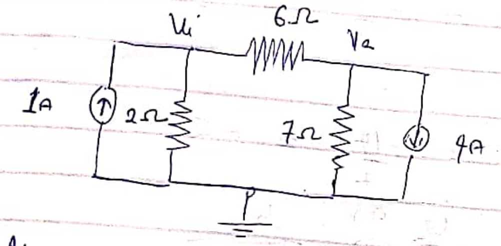
$V_1 = \frac{144 - 6V_2}{-4}$

$V_1 = 0$

$V_1 = 0V$, $V_2 = 24V$

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

iii) Obtain V_1 and V_2 and the currents through the resistors the circuit in example (ii) if the 2A current source was replaced by a 1A current source.



At node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V$

$6 = 4V_1 - V_2$ --- (i)

At node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ --- (ii)

from eqn (i), $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eqn (1)

$108 = 7V_1 - 13(4V_1 - 6)$

$108 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Subs $V_1 = -2$ in eqn (1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$V_1 = -2V, \quad V_2 = -14V$

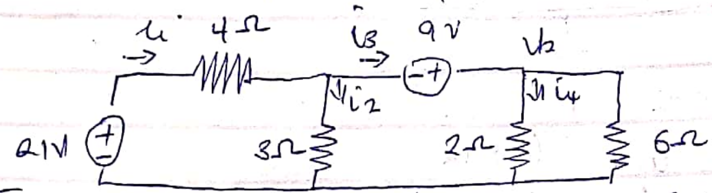
Current through the resistors

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

(3)



Find the Current through the 3Ω and 2Ω resistors

Using KCL at node V_1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0$ — (i)

Using KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ --- (ii)

$7V_1 + 8V_2 = 63$ — (i)

$-V_1 + V_2 = 9$ — (ii)

Let $V_2 = 9 + V_1$

Sub $V_2 = 9 + V_1$ in eqn (4)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V_1 = 0.6V$

Sub $V_1 = 0.6$ in eqn (1)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

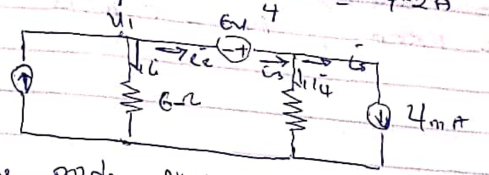
$V_2 = 8.4V$

$V_1 = -0.6V$ and $V_2 = 8.4V$

NK ∴ Current through the 2Ω resistor

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4)



Find the node voltage and the current through the 6Ω and 12Ω resistor

Let Assume that $V_1 - V_2 = 6V \Rightarrow i_2$

At node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ — (i)

At node 2

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2$ — (ii)

Scanned with CamScanner

Solving V_1 and V_2 Simultaneously we ha
$V_1 = 9.5V$ and $V_2 = 5.1V$

Current through the 6Ω resistor

$i = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$, $i_2 = V_1 - V_2$

$= 9.5 - 5.1 = 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V$, $V_2 = 5.1V$
$i = 1.58A$, $i_4 = 0.43A$