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17/ENG04/031
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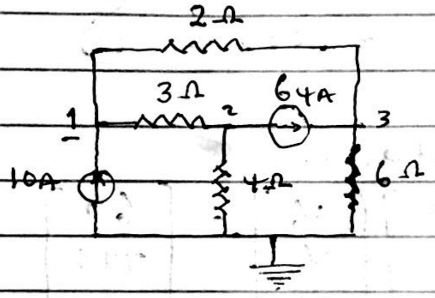
CIRCUIT ~~LINEAR~~ THEORY.

Question 1

Find the Voltages at nodes 1, 2 and 3 in the circuit below.



At node 1, KCL

$$10 = i_1 + i_2 \rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2.$$

$$60 = 5V_1 - 2V_2 + 2V_1 \quad \text{— (i)}$$

At node 2, KCL

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2 - 0$$

$$768 = 4V_1 - 7V_2 \quad \text{— (ii)}$$

At Node 3, KCL

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ —— (iii)

Using Crammer's Rule.

$5V_1 - 2V_2 - 3V_3 = 60$ --- (1)

$4V_1 - 7V_2 = 748$ --- (2)

$-3V_1 + 4V_3 = 384$ --- (3)

In Matrix Representation.

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 748 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$ & $V_3 = \frac{\Delta_3}{\Delta}$.

where $\Delta = \begin{vmatrix} \overset{+}{5} & \overset{-}{-2} & \overset{+}{-3} \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} \overset{+}{60} & \overset{-}{-2} & \overset{+}{-3} \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$= 60(-28-0) - 768(-8-0) - 384(0$

$= -1680 + 6144 - 8064.$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$$V_2 = \Delta_2 = \begin{matrix} + \\ - \\ + \end{matrix}\begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$$= 5(768 - 0) - 4(240 + 1152) - 3(0 + 2244)$$

$$= 2880$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

For V_3,

$$\begin{matrix} + \\ - \\ + \end{matrix}\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$$

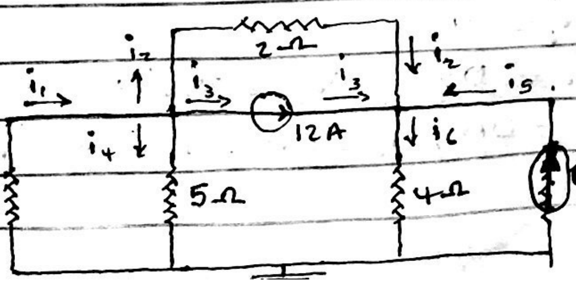
$$= -7020$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$$

$\therefore$ $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

Question 2

2. Find the voltages at nodes 1 and 2 and determine the current flowing through the four resistors in the circuit below.



At Node 1, KCL

$i_1 = i_2 + i_3 + i_4$.

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \quad \text{— (i)}$$

At Node 2,

$i_3 + i_2 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2(V_2)$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \quad \text{— (ii)}$$

Using Elimination Method:

$$120 = -8V_1 + 5V_2 \quad \times 2$$

$$144 = -4V_1 + 6V_2 \quad \times 4$$

$$240 = -16V_1 + 10V_2 \quad \text{— (iii)}$$

$$576 \quad -16V_1 + 24V_2 \quad \text{— (iv)}$$

$$816 \quad 0 + 34V_2$$

$$V_2 = 24V$$

Sub V_2 in eqn (iii)

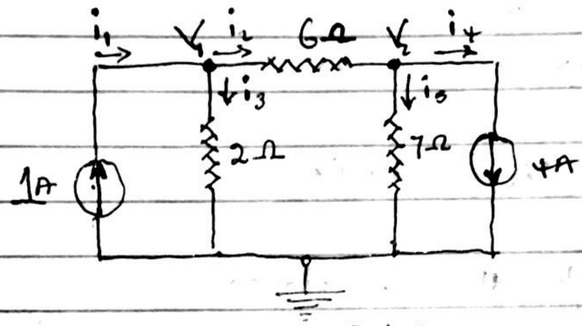
$$240 = -16V_1 + 240$$

$$V_1 = 0$$

$\therefore$ $V_1 = 0V$, $V_2 = 24V$.

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$.

2ii Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current 2A.



Solution

At Node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = (V_1 - V_2) + 3V_1$$

$$6 = 4V_1 - V_2 \quad \text{---} \ (i)$$

At Node 2.

$$i_2 = i_4 + i_5$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \quad \text{---} \ (ii)$$

From eqn (i), $V_2 = 4V_1 - 6$

Sub V_2 in eqn (ii).

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$168 - 78 = 7V_1 - 52V_1$$

$$90 = -45V_1$$

$$V_1 = -2V.$$

Sub $V_1 = -2$ in eqn (i)

~~$6 = 4V_1$.~~

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$= -14V$.

$\therefore V_1 = -2V, \ V_2 = -14V$.

Current through the resistors:

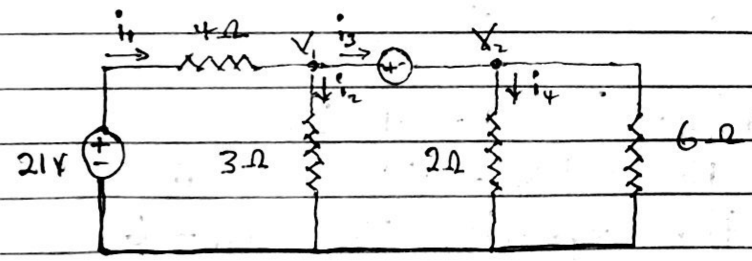
$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$.

Question 3

3)



Find the current through the 3Ω and 2Ω resistors.

Solution

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \text{— (i)}$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \text{— (ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{— } \times 1$$

$$-V_1 + V_2 = 9 \quad \text{— } \times 7$$

$$7V_1 + 8V_2 = 63 \quad \text{— (iii)}$$

$$+ \quad -7V_1 + 7V_2 = 63 \quad \text{— (iv)}$$

$$0 + 15V_2 = 126$$

$$V_2 = \frac{126}{15} = 8.4V$$

Sub V_2 in eqn (iv)

$$-7V_1 + 7(8.4) = 63$$

$$-7V_1 = 63 - 58.8$$

$$-7V_1 = 4.2$$

$$V_1 = -0.6V.$$

Current through the 2Ω resistor,

$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A.$$

Current through the 2Ω resistor,

$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A.$$

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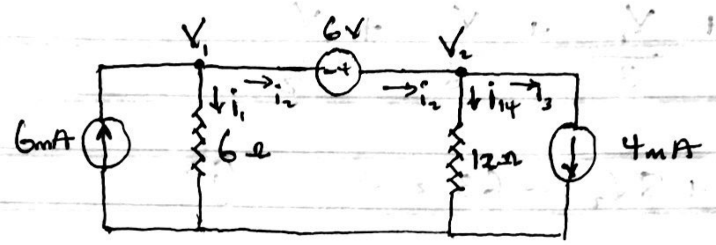
Question. 4

4.)



Find the node Voltages and the currents through the 6Ω and 12Ω resistor.

Solution

let say $V_1 - V_2 = 6V \rightarrow i_2$

At node 1, Using KCL.

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$0.036 = V_1 + 6(V_1 - V_2)$$

$$0.036 = V_1 + 6V_1 - 6V_2$$

$$0.036 = 7V_1 - 6V_2 \quad \text{— (i)}$$

At Node 2.

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 0.048 + V_2$$

$$0.048 = 12V_1 - 12V_2 - V_2$$

$$0.048 = 12V_1 - 13V_2 \quad \text{— (ii)}$$

$$7V_1 - 6V_2 = 0.036 \quad \times 12 \quad \text{— 1}$$

$$12V_1 - 13V_2 = 0.048 \quad \times 7 \quad \text{—}$$

$$84V_1 - 72V_2 = 0.432 \quad \text{— (iii)}$$

$$84V_1 - 91V_2 = 0.336 \quad \text{— (iv)}$$

$$0 + 19V_2 = 0.096$$

$$V_2 = 5.053 \times 10^{-3} V$$

Sub V_2 in eqn (iv)

$$84V_1 - 0.4598 = 0.336$$

$$84V_1 = 0.336 + 0.4598$$

$$V_1 = \frac{0.7958}{84} = 9.474 \times 10^{-3} \, V$$

∴ Current through the 6 Ω resistor,

$$i_1 = \frac{V_1}{6} = \frac{9.474 \times 10^{-3}}{6} = 1.579 \times 10^{-3} \, A,$$

Current through the 12 Ω resistor,

$$i_4 = \frac{V_2}{12} = \frac{5.053 \times 10^{-3}}{12} = 4.212 \times 10^{-4} \, A$$

$V_1 = 9.474 \times 10^{-3} \, V$

$V_2 = 5.053 \times 10^{-3} \, V$

$i_1 = 1.579 \times 10^{-3} \, A$

$i_4 = 4.212 \times 10^{-4} \, A.$