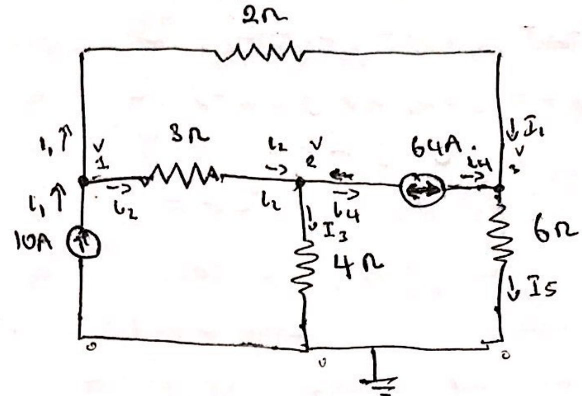
Obot Mkpouto Obot
17/ENG04/046
Elect/Elect
Circuit theory assignment

1) Find the voltages at node 1, 2 and 3 in The circuit below



At node 1 using KCL

$10A = i_1 + i_2$

$10A = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

multiply by 6

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_3$ — ①

At Node 2, using KCL

$i_2 = I_3 + I_4$

$i_2 = I_3 + 64$

$64 = I_2 - I_3$

$64 = \frac{V_1 - V_3'}{3} - \frac{V_2 - 0}{4}$

multiply by 12

Scanned with CamScanner

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2 + 0$

$768 = 4V_1 - 7V_2$ — ②

At Node 3 kcl

$64 = -I_1 + I_5$

$64 = I_5 - I_1$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

multiply 6

$384 = V_3 - 3(V_1 - V_3)$

$384 = V_3 - 3V_1 + 3V_3$

$384 = -3V_1 + 4V_3$ — ③

using Crammers rul

$5V_1 - 2V_2 - 3V_3 = 60$ — ①

$4V_1 - 7V_2 + 0 = 768$ — ②

$-3V_1 + 0 + 4V_3 = 384$ — ③

matrix :

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & -4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$ $V_3 = \frac{\Delta_3}{\Delta}$

where

Scanned with CamScanner

Where Δ

$$\begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5((-7\times4)-(0)) - (-2)((4\times4)-(0\times-3)) - 3((4\times0)-(-7\times-3))$

$= 5(-28) + 2(16) - 3(-21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 \quad \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

~~$= 60((-7\times4)-(0)) - (-2)((768\times4)-(384\times0))$~~

$= 60((-7\times4)-(0)) - 768((-2\times4)-(-3\times0))\ 384((-2\times0)-(-3\times-7))$

$= 60(-28) + 768(-8) + 384(0-(21))$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{-3600}{-48} = 80V$

~~$$\Delta_2 : \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$~~

~~$= 60((4\times4)-(-3\times0)) - 768((5\times4)-(-3\times-3)) + 384((5\times0)-(-3\times3))$~~

Since $V_1 = 80V$.

we have: from eq ① and ③

$8V_1 - 2V_2 - 3V_3 = 60$ — ①

$-3V_1 + 4V_3 = 384$ — ③

replacing V_1 with 80

$8(80) - 2V_2 - 3V_3 = 60$ ..... (i)

$-3(80) + 4V_3 = 384$ ..... (ii)

$400 - 2V_2 - 3V_3 = 60$ ..... (iii)

$-240 + 4V_3 = 384$ ..... (iv)

$2V_2 - 3V_3 = -340$ — (v)

$4V_3 = 624$ --- (vi)

using (vi) to find for V_3

$V_3 = \frac{624}{4}$

$V_3 = 156V$

putting V_3 in (v)

$-2V_2 - 3V_3 = -340$

$-2V_2 - 3(156) = -340$

$-2V_2 - 468 = -340$

$-2V_2 = 128$

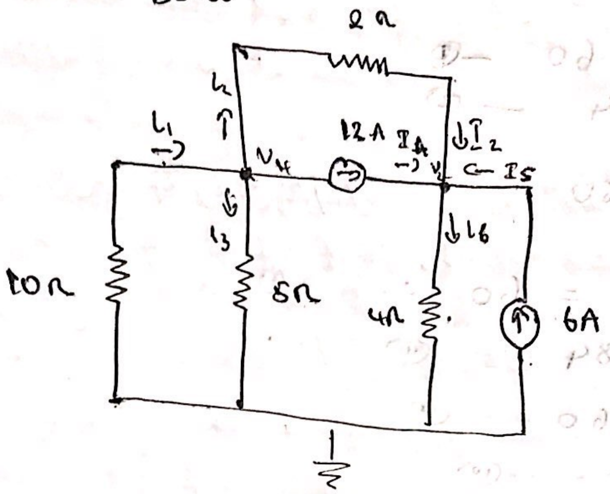
$-V_2 = 64V$

$V_2 = -64V$

Voltages are
$V_1 = 80V$
$V_2 = -64V$
$V_3 = 156V$

Scanned with CamScanner

2). Find the voltages at node 1 and 2 and determine the currents flowing through the four resistors in circuit below



At Node 1 with KCL

$i_1 = I_2 + I_3 \quad I_4$

~~$I_A = \frac{V_1 - V_2}{2}$~~

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + \frac{V_1 - V_0}{5} + 12A$$

multiply through by 10

$V_0 - V_1 = 5(V_1 - V_2) + 2(V_1 - V_0) + 120$

$V_0 - V_1 = 5V_1 - 5V_2 + 2V_1 - 2V_0 + 120$

$0 = 6V_1 - 5V_2 + 2V_1 + 120$

$0 = 8V_1 - 5V_2 + 120$

$-8V_1 + 5V_2 = 120$ —(1)

At node 2

$I_A = -I_2 - I_5 + I_6$

$I_6 = I_4 + I_2 + I_5$

$$\frac{V_2 - V_0}{4} = 12 + \frac{V_1 - V_2}{2} + 6V$$

multiply by 4

$V_2 - V_0 = 48 + 2(V_1 - V_2) + 24$

$V_2 = 48 + 2V_1 - 2V_2 + 24$

$V_2 = 72 + 2V_1 - 2V_2$

$0 = 72 + 2V_1 - 3V_2$

$+3V_2 - 2V_1 = 72$ —— ②

Using elimination method

$-8V_1 + 5V_2 = 120$ —— ① $\times -4$

$-2V_1 + 3V_2 = 72$ —— ② — -8

~~$+32V_1 - 200V_2 = -480$ — ①~~

~~$-\ 10V_1 - 24V_2 = -576$ — ②~~

~~$+2V_1 + 4V_2 = 96$~~

$+16V_1 \mp 10V_2 = -240V$

$-\ +16V_1 - 24V_2 = -576V$

$0 + 14V_2 = 336$

$V_2 = 24V$

Sub 24V in equation ①

$-8V_1 + 5(24) = 120V$

$-8V_1 + 120 = 120V$

$-8V_1 = 2400$

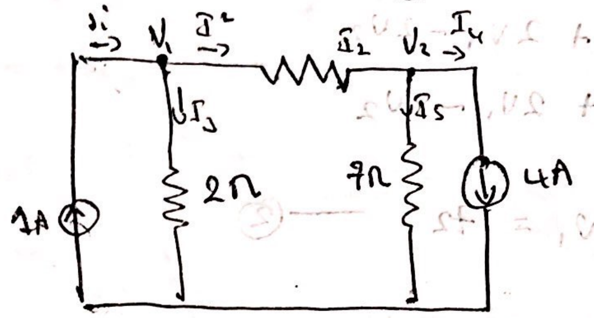
Scanned with CamScanner

$V_1 = \cancel{400}\ 0V$

$V_1 = 0V$, $V_2 = 24V$

$I_1 = 0A$, $I_2 = 0A$, $I_3 = 6A$, $I_4 = 12A$

III) obtain V_1 and V_2 and the Currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by ~~2A~~ 1A current source



At Node 1

$I_1 = I_2 + I_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ --- ①

At Node 2

$I_2 = I_3 + I_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ --- ②

from eq ① $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eqn ②

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Subs $V_1 = -2$ in eqn ①

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -14V$

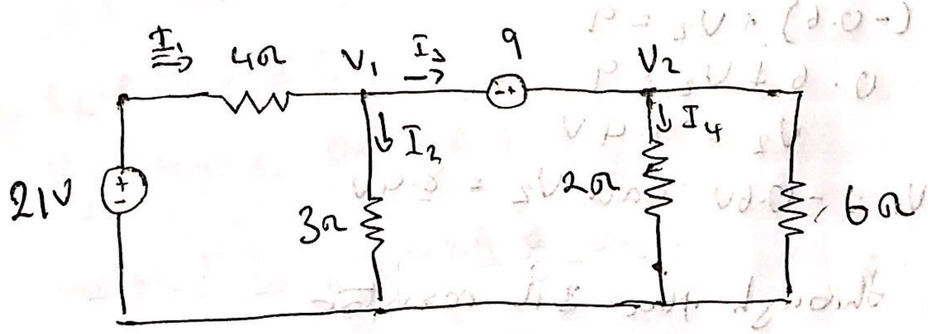
∴ Current through the resistor

$I_1 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$

$I_2 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$I_3 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

③



Find the current through the 3Ω and 2Ω resistor

using KCL at Node 1

$I_1 + I_2 + I_3 + I_4 = 0$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$7V_1 + 8V_2 - 63 = 0 \quad \ldots (i)$

Using KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9 \quad \ldots\ldots (ii)$

$7V_1 + 8V_2 = 63 \quad —— (i)$

$V_1 + V_2 = 9 \quad —— (ii)$

let $V_2 = 9 + V_1$

Sub $V_2 = 9 + V_1$ in eqn (i)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V_1 = -0.6V$

Sub $V_1 = -0.6$ in eqn (ii)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 8.4V$

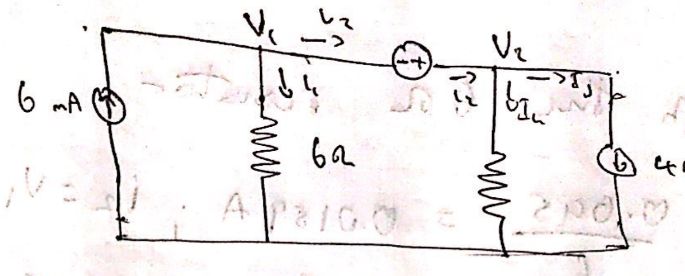
$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

Current through the 3Ω resistor

$$I_2 = \frac{V_1}{3} = -\frac{0.6}{3} = -0.2A$$

Current through the 2Ω resistor

$$I_1 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$$



Find the node voltage and the currents through the 6Ω and 12 Ω resistor.

let Assume that $V_1 - V_2 = 6V \Rightarrow I_2$

At node 1 using k.c.l

6mA = 0.06 A

$0.06A = I_1 + I_2$

$0.06A = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$0.36 = V_1 + 6(V_1 - V_2)$

$0.36 = V_1 + 6V_1 - 6V_2$

$0.36 = 7V_1 - 6V_2$ ------ (i)

At node 2

$I_2 = I_3 + I_4$

$V_1 - V_2 = 0.04A + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 0.48A + V_2$

$0.48A = 12V_1 - 12V_2 - V_2$

$0.48A = 12V - 13V_2$ ------ (ii)

Solving V_1 and V_2 simultaneously we get:

$V_1 = 0.095$

$V_2 = 0.05V$

Scanned with CamScanner

Current through the 6 Ω resistor

$i_1 = \frac{V_1}{6} = \frac{0.095}{6} = 0.0159\,A$; $i_2 = V_1 - V_2 =$

~~9.5 - 5.1 = 4.4A~~ $0.095 - 0.051V =$

$= 0.044A$

Current through the 12 Ω resistor

$i_4 = \frac{V_2}{12} = \frac{0.051}{12} =$ ~~0.428~~ $0.00425\,A$

$V_1 = 0.095$, $V_2 =$ ~~8~~ $0.051V$

$i_4 = 0.00425$ $i_1 = 0.0159A$