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MATRIC NO: 18/SCI01/082

**Question**

1.  Define a vector space.

2. Show that the vectors  A= (1,1, 1), B = (1, 2, 3,), C  = (1,5,8)  spans R3.

3.  Are the vectors P= (1,2, 3),  Q= (3, 2, 1), R = (0,0,1)  a basis for R3 ?

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**SOLUTION**

1. A vector space is a set that is closed under finite vector addition and scalar multiplication.
2. $αA+βB+γC=(a,b,c)$

α 1 + β 1 + γ 1

 1 2 5

 1 3 8

α + β + γ= a …. (i)

α + 2β + 5γ= b …. (ii)

α + 3β + 8γ= c …. (iii)

from equation (i)

α + β + γ= a

α = a - β – γ ….(iv)

putting equation (iv) in 2&3

from equation 2

α + 2β + 5γ= b

(a - β – γ )+ 2β + 5γ= b

a - β + 2β + 5γ– γ= b

β + 4γ = b – a …..(v)

from equation 3

α + 3β + 8γ= c

(a –β –γ) + 3β + 8γ= c

a – β + 3β – γ + 8γ= c

2β + 7γ = c-a …..(vi)

Combining equation (v) & (vi)

β + 4γ = b – a …x2

2β + 7γ = c-a ….x1

 2β + 8γ =2 b –2 a

- 2β + 7γ = c-a

 γ = (2b-2a)-(c-a)

 γ = 2b-2a+c+a

  **γ = -a + 2b – c**

from equation (v)

β + 4γ = b – a

β + 4(-a + 2b - c) = b – a

β – 4a + 8b -4c = b – a

β = 4a - 8b + 4c + b – a

**β= 3a – 7b + 4c**

from equation (iv)

 α = a - β – γ

α = a – (3a -7b + 4c) – (-a + 2b –c)

α = a – 3a +7b -4c + a -2b +c

**α = -a + 5b -3c**

$∴ αA+βB+γC $

**= (**$-a+5b-3c)A+\left(3a-7b+4c\right)B-\left(a+2b-c\right)C$

1. For linear independence

αP +βQ + γR= 0

α 1 + β 3 + γ 0 0

 2 2 0 = 0

 3 1 1 0

 α + 3β + 0 0

 2α 2β 0 = 0

 3α β γ 0

 α + 3β = 0 …..i

 2α + 2β = 0.....ii

 3α + β + γ= 0 ……iii

From equation (i)

 α + 3β = 0

 α= -3β …. (iv)

From (ii)

 2α + 2β = 0

 2(-3β) + 2β = 0

 -6β+ 2β=0

 -4β=0

 **β= 0**

from (iv)

 α= -3β

 α= -3(0)

 α= 0

since **α= 0 , β= 0 & y=0** the vectors are linearly independent

for spanning set

 α + 3β = a …..i

 2α + 2β = b .....ii

 3α + β + γ= c ……iii

From equation (i)

α + 3β = a

α = a - 3β …..(iv)

putting (iv) in ii & iii

2α + 2β = b

2(a - 3β) + 2β = b

2a - 6β + 2β=b

 -4β= b -2a

 **β=** $\frac{-b+2a}{4}$

putting β in equation (iv)

 α = a - 3β

α = a – 3$(\frac{-b+2a}{4})$

 = $\frac{a}{1}+\frac{ 3b-6a}{4}$

= $\frac{4a+3b-6a}{4}$

**α=** $\frac{-2a+3b}{4}$

from equation (iii)

3α + β + γ= c

3 $(\frac{-2a+3b}{4})$ + $(\frac{-b+2a}{4})$ + γ= c

 $\frac{-6a+9b}{4}-\frac{b+2a }{4}- \frac{c}{1}= -γ $

$$=\frac{-6a+9b- \left(-b+2a\right)-4c}{4}= - γ$$

$= \frac{-6a+9b-b+2a+4c}{4}$ = -$ γ$

$$γ= a-2b+c$$

**Therefor P, Q, R are basis for R3**