

at node V_1

$$i_3 = i_1 + i_5 + i_6 = 12 + i_5 + i_6$$

$$\therefore 12 = i_3 - i_5 - i_6$$

$$12 = \frac{V_2 - V_1}{2} - \left[\frac{V_1 - 0}{5} \right] - \left[\frac{V_1 - 0}{10} \right]$$

$$12 = \frac{V_2 - V_1}{2} - \frac{V_1}{5} - \frac{V_1}{10}$$

$$12 = \frac{5(V_2 - V_1) - 2(V_1) - V_1}{10}$$

$$120 = 5V_2 - 5V_1 - 2V_1 - V_1$$

$$120 = 5V_2 - 8V_1 \quad \text{--- (1)}$$

From node V_2

$$i_1 + i_2 = i_3 + i_4$$

$$12 + 6 = \frac{V_2 - V_1}{2} + \left[\frac{V_2}{4} \right]$$

$$18 = \frac{V_2 - V_1}{2} + \frac{V_2}{4}$$

$$18 = \frac{2(V_2 - V_1) + V_2}{4}$$

$$72 = 3V_2 - 2V_1 \quad \text{--- (2)}$$

Using Simultaneous Eqn

$$120 = 5V_2 - 8V_1 \quad \text{--- } \times (2)$$

$$72 = 3V_2 - 2V_1 \quad \text{--- } \times (5)$$

$$240 = 10V_2 - 16V_1$$

$$\Rightarrow \quad - \quad 576 = 24V_2 - 16V_1$$

$$\hline -336 = -14V_2$$

$$V_2 = 24V$$

From Eqn (1)

$$120 = 5(24) - 8V_1$$

$$120 = 120 - 8V_1$$

$$8V_1 = 120 - 120$$

$$V_1 = 0V$$

Current flowing through the 2Ω resistor

$$= \frac{V_2 - V_1}{2} = \frac{24 - 0}{2} = 12A$$

while current flowing through the 4Ω resistor

$$= \frac{V_2}{4} = \frac{24}{4} = 6A$$

Due to V_1 being 0V then current passing through the 10Ω and 5Ω resistors both equals zero (0).

ii) V_1 and V_2 when $12A$ is replaced with $1A$ at node V_1

$$1 = \frac{5(V_2 - V_1) - 2(V_1) - V_1}{10}$$

$$10 = 5V_2 - 8V_1 \quad \text{--- (1)}$$

at node V_2

$$1 + 6 = \frac{V_2 - V_1}{2} + \frac{V_2}{4}$$

$$7 = \frac{2(V_2 - V_1) + V_2}{4}$$

$$28 = 3V_2 - 2V_1 \quad \text{--- (2)}$$

Using Simultaneous Eqn

$$10 = 5V_2 - 8V_1 \quad \text{--- } \times (2)$$

$$28 = 3V_2 - 2V_1 \quad \text{--- } \times (5)$$

$$\Rightarrow \quad 20 = 10V_2 - 16V_1$$

$$- \quad 224 = 24V_2 - 16V_1$$

$$\hline -204 = -14V_2$$

$$\therefore V_2 = 14.57$$

and by extension using eqn (1)

$$10 = 5(14.57) - 8V_1$$

$$10 = 72.86 - 8V_1$$

$$8V_1 = 62.86$$

$$V_1 = 7.86V$$

Current through the 2Ω resistor

$$= \frac{V_2 - V_1}{2} = \frac{14.57 - 7.86}{2}$$

$$= 3.355A$$

Current through the 4Ω resistor

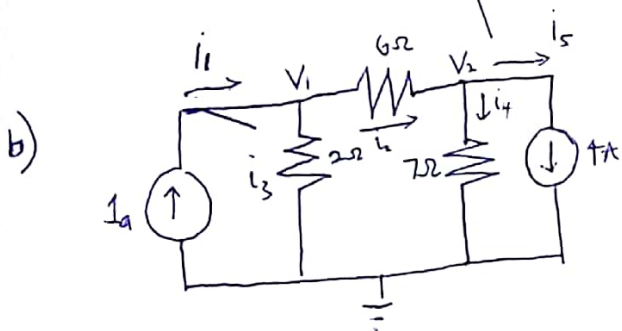
$$= \frac{V_2}{4} = \frac{14.57}{4} = 3.6425A$$

Current through the 10Ω resistor

$$= \frac{V_1}{10} = \frac{7.86}{10} = 0.786A$$

Current through the 5Ω resistor

$$= \frac{V_1}{5} = \frac{7.86}{5} = 1.572A$$



at node 2

$$i_1 = i_2 + i_3$$

$$1 = \left[\frac{V_1 - V_2}{6} \right] + \left[\frac{V_1 - 0}{2} \right]$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$1 = \frac{V_1 - V_2 + 3V_1}{6}$$

$$6 = 4V_1 - V_2 \quad (1)$$

at node 2

$$i_2 = i_4 + i_5$$

$$\left[\frac{V_1 - V_2}{6} \right] = \left[\frac{V_2}{7} \right] + 4$$

$$\frac{V_1 - V_2}{6} - \frac{V_2}{7} = 4$$

$$\frac{7(V_1 - V_2) - 6V_2}{42} = 4$$

$$108 = 7V_1 - 13V_2 \quad (2)$$

Solving Simultaneously

$$V_1 = -2V \text{ and } V_2 = 14V$$

From this Current moving through the 6Ω

$$= \frac{V_1 - V_2}{6}$$

$$= \frac{-2 - 14}{6} = \frac{-16}{6}$$

$$= -2.67A$$

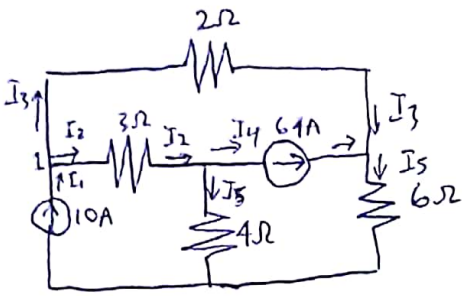
From this Current moving through the 2Ω resistor

$$= \frac{V_1}{2} = \frac{-2}{2} = 1A$$

Current moving through 7Ω

$$= \frac{V_2}{7} = \frac{14}{7} = 2A$$

QED



From node 1

$$I_1 = I_2 + I_3$$

$$\Rightarrow 10 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2}$$

$$10 = \frac{(V_1 - V_2)2 + (V_1 - V_3)3}{6}$$

$$60 = 2V_1 - 2V_2 + 3V_1 - 3V_3$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{--- (1)}$$

From node 2

$$I_2 = I_5 + I_4$$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{4} + 6$$

$$\frac{V_1 - V_2}{3} - \frac{V_2}{4} = 6$$

$$\frac{4V_1 - 4V_2 - 3V_2}{12} = 6$$

$$\frac{4V_1 - 7V_2}{12} = 6$$

$$4V_1 - 7V_2 = 768 \quad \text{--- (2)}$$

From node 3

$$I_4 + I_3 = I_5$$

$$\cancel{64} + \frac{V_3}{4} = \frac{V_3}{6}$$

$$\cancel{64} = \frac{V_3}{6} - \frac{V_3}{4}$$

$$\cancel{\frac{2V_3 - 3V_3}{4} = 64}$$

$$\cancel{768 = 2V_3 + 3V_3}$$

$$64 = \frac{V_1 - V_3}{2} = \frac{V_3}{6}$$

$$64 = \frac{V_3}{6} - \frac{(V_1 - V_3)}{2}$$

$$64 = \frac{2V_3 - 3V_1 + 3V_3}{6}$$

$$\cancel{64 = \frac{(6V_1 + 4V_3)}{6}}$$

$$\cancel{768 = 6V_1 + 4V_3}$$

$$384 = 4V_3 - 3V_1 \quad \text{--- (3)}$$

$$60 = 5V_1 - 2V_2 - 3V_3$$

$$768 = 4V_1 - 7V_2 + 0V_3$$

$$384 = -3V_1 + 0V_2 + 4V_3$$

Using Cramer's Rule

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}; \quad V_2 = \frac{\Delta_2}{\Delta}; \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$$= 5 \begin{vmatrix} -7 & 0 \\ 0 & 4 \end{vmatrix} + 2 \begin{vmatrix} 4 & 0 \\ -3 & 4 \end{vmatrix} - 3 \begin{vmatrix} 4 & -7 \\ -3 & 0 \end{vmatrix}$$

$$= 5(-28 - 0) + 2(16 - 0) - 3(0 - 21)$$

$$= -140 + 32 + 63 = -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$$= 60(-28) + 2(3072 - 0) - 3(0 + 2688)$$

$$= -1680 + 6144 - 8064$$

$$= 6144 - 9744$$

$$= -3600$$

$$\therefore V_1 = \frac{-3600}{-45} = 80 \text{ V}$$

$$\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$$= 5(3072 - 0) - 60(16 - 0) - 3(1536 + 2304)$$

$$= 15360 - 960 - 11520$$

$$= 15360 - 12480$$

$$= 2880$$

$$\therefore V_2 = \frac{2880}{-45} = -64 \text{ V}$$

$$\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5(-2688 - 0) + 2(1536 + 2304) + 60(0 - 21)$$

$$= -13440 + 7680 - 1260 = -7020$$

$$\therefore V_3 = \frac{-7020}{-45} = 156 \text{ V}$$