

Assignment

At node 1, KCL

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \dots (i)$$

Node 2, KCL

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \quad \dots (ii)$$

At node 3, KCL

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_2}{2}$$

$$384 = V_3 - 3(V_1 - V_2)$$

$$384 = -3V_1 + 4V_3 \quad \dots (iii)$$

Using Cramer's Rule

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \dots (i)$$

$$4V_1 - 7V_2 = 768 \quad \dots (ii)$$

$$-3V_1 + 4V_3 = 384 \quad \dots (iii)$$

Matrix Representation.

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\text{where } \Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$$\begin{aligned} &= 5(-2 \cdot 8 - 0) + 2(16 + 0) - 3(0 - 21) \\ &= -140 + 32 + 63 \\ &= -45 \end{aligned}$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-280) - 768(-80) + 384(0 - 21)$$

$$= -1680 + 6144 - 8064$$

$$= -3600$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$$

$$\text{For } V_2: \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$$= 5(768 \times 4 - 0) - 4(60 \times 4) - (384 \times 3) - 3(0 - (768 \times 3))$$

$$= 2880$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

$$\text{For } V_3: \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5(-7 \times 384 - 0) - 4(2 \times 384 - 0) - 3(-2 \times 768) - (-7 \times 60)$$

$$= -7020$$

$$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$$

$$\text{Hence } V_1 = 80V, \quad V_2 = -64V, \quad V_3 = 156V$$

At node I, KCL

$$i_1 = i_2 + i_3 + i_4$$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

Multiply through by 10

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \quad \dots (i)$$

At Node 2

$$i_3 + i_2 + i_5 = i_6$$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2(V_2)$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \quad \dots (ii)$$

Using Elimination method

$$120 = -8V_1 + 5V_2 \quad \dots (i) \times -4$$

$$144 = -4V_1 + 6V_2 \quad \dots (ii) \times -8$$

$$-480 = 32V_1 - 20V_2 \quad \dots (iii)$$

$$-1152 = 32V_1 - 48V_2 \quad \dots (iv)$$

Subtract eqn (iii) from (iv)

$$-672 = 0 - 28V_2$$

$$V_2 = \frac{-672}{-28}$$

$$V_2 = 24$$

Sub. $V_2 = 24$ in eqn (ii)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6V_2}{-4}$$

$$V_1 = \frac{144 - 6 \times 24}{-4}$$

$$V_1 = 0$$

$$V_1 = 0V, V_2 = 24V$$

$$i_1 = 0A, i_2 = 0A, i_3 = 6A, i_4 = -12A$$

ii)

At node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \quad \text{--- (i)}$$

At node 2

$$i_2 = i_4 + i_5$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 + 3V_2 \quad \text{--- (ii)}$$

from eqn (i); $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eqn (ii)

$$168 = 7V_1 + 3(4V_1 - 6)$$

$$168 = 7V_1 + 12V_1 - 18$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45}$$

$$V_1 = -2V$$

Subs $V_1 = -2$ in eqn (i)

$$6 = 4(-2) - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = -14V$$

$$V_1 = -2V, V_2 = -14V$$

Current through the resistors;

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_3 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$

③ Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \dots (i)$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \dots (ii)$$

$$7V_1 + 8V_2 = 63 \quad \dots (i)$$

$$-V_1 + V_2 = 9 \quad \dots (ii)$$

Let $V_2 = 9 + V_1$

Sub $V_2 = 9 + V_1$ in eqn (i)

$$7V_1 + 8(9 + V_1) = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = -9$$

$$V_1 = -0.6V$$

Sub $V_1 = -0.6$ in eqn (ii)

$$-(-0.6) + V_2 = 9$$

$$0.6 + V_2 = 9$$

$$V_2 = 8.4V$$

$$V_1 = -0.6V \text{ and } V_2 = 8.4V$$

Current through the 3Ω resistor;

$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2Ω resistor.

$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 2.1A$$

4) At node 1 using KCL

$$i_{mA} = i_1 + i_2$$

$$4mA = \frac{V_1 - 0}{6} + \frac{V_1 - V_2}{12}$$

$$36 = V_1 + 6(V_1 - V_2)$$

$$36 = 7V_1 - 6V_2 \quad (i)$$

At node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 48 + V_2$$

$$48 = 12V_1 - 12V_2 - V_2$$

$$48 = 12V_1 - 13V_2 \quad (ii)$$

Solving V_1 and V_2 simultaneously, we have

$$V_1 = 9.5V \text{ and } V_2 = 5.1V$$

∴ Current through the 6-Ω resistor

$$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A \quad i_2 = \frac{V_1 - V_2}{12} = \frac{9.5 - 5.1}{12} = 4.4A$$

Current through the 12-Ω resistor

$$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$$

$$V_1 = 9.5V, \quad V_2 = 5.1V$$

$$i_1 = 1.58A, \quad i_4 = 0.43A //$$