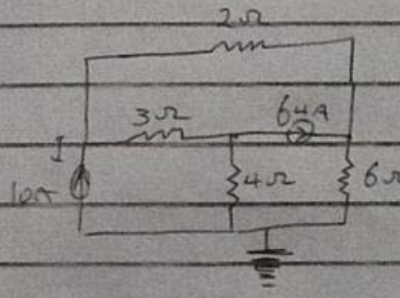
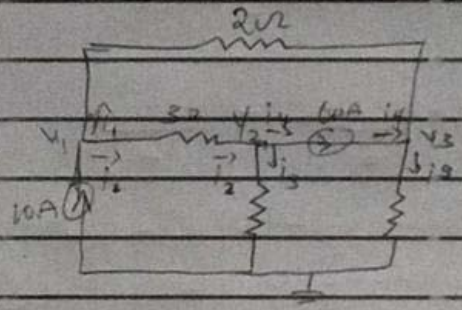
Circuit Theory Answers

① Find the Voltages at node 1, 2, and 3 in the Circuit below





~~At~~ Node 1 KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

Multiply through by 6

$60 = 3[V_1 - V_3] + 2[V_1 - V_2]$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

~~$60 = 5V_1 - 5V_2$~~

$60 = 5V_1 - 3V_3 - 2V_2 \quad -0$

At Node 2 KCL

$i_2 = i_3 + 6A$

$6A = i_2 - i_3$

$6A = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

#

Multiply through by 7

$768 = 4[V_1 - V_2] - 3[V_2 - 0]$

$768 = 4V_1 - 7V_2$ — (2)

Node 3 KCL

$64 + i_1 = i_5$

$64 = i_5 - i_4$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3[V_1 - V_3]$

$384 = -3V_1 + 4V_3$ — (3)

$5V_1 - 2V_2 - 3V_3 = 60$ — (1)

$4V_1 - 7V_2 + 0V_3 = 768$ — (2)

$-3V_1 + 0V_2 + 4V_3 = 384$ — (3)

Matrix Rep.

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$ $V_2 = \frac{\Delta_2}{\Delta}$ $V_3 = \frac{\Delta_3}{\Delta}$

#

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5[-28-0] + 2[16+0] - 3[0-21]$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60[-28-0] - 768[-8-0] - 384[0-21]$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$= 5[(768 \times 4) - 0] - 4[(60 \times 4) - (384 - 3)] - 3[0 - (768 - 3)]$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$V_3 = \frac{\Delta_3}{\Delta}$ ~~= 0~~

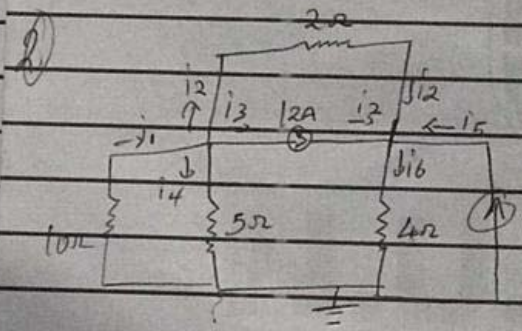
$$V_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5[(-7\times384)-0] - 4[(-2\times384)-0] - 3[(-2\times768)-(-7\times60)]$

$= -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

$\therefore V_1 = 80V \quad V_2 = -64V \quad V_3 = 156V$



Node 1 KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_3}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5[V_1 - V_2] + 120 + 2[V_1 - 0]$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \text{--- (1)}$

#

#

Node 2.

$I_3 + I_2 + i_5 = 16$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4[V_1 - V_2] + 48 = 2[V_2]$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \quad -(2)$$

Using Elimination method-

$$120 = -8V_1 + 5V_2 \quad -(1) \times 4$$

$$144 = -4V_1 + 6V_2 \quad -(ii) \times 8$$

$$-480 = 32V_1 - 20V_2 \quad \cdots (iii)$$

$$-1152 = 32V_1 - 48V_2 \quad \cdots (iv)$$

Subtract eqn 3 from 4

$$-672 = 0 - 28V_2$$

$$-672 = -28V_2$$

$$V_2 = \frac{-672}{-28}$$

$V_2 = 24V$

Sub V_2 into eqn (ii)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6[24]}{-4}$$

$$V_1 = \frac{144 - 144}{-4} \quad V_1 = 0V$$

#

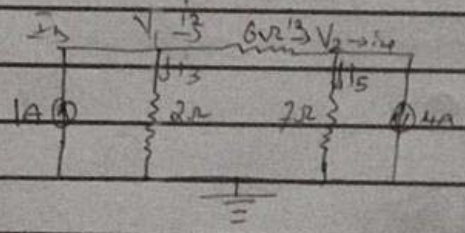
$\therefore V_1 = 0V \quad V_2 = 24V$.

$i_1 = 0A, \; i_3 = 0A, \; i_2 = 6A \; i_4 = -12A$

(2) Obtain V_1 and V_2 and the currents through the resistors

Classroom

(2)



Node 1

$i_1 = i_a + i_2$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ - (1)

Node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7[V_1 - V_2] = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ - (2)

From equation (1)

$V_2 = 4V_1 - 6$

Sub $V_2 = 4V_1 - 6$ into eqn (2)

$168 = 7V_1 - 13[4V_1 - 6]$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Subs $V_1 = -2$ into eqn 1

$6 = 4[-2] - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

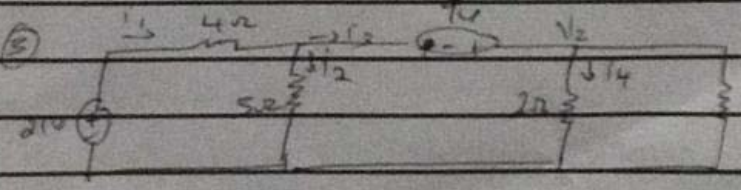
$V_1 = -2V$; $V_2 = +14V$

Currents through the resistors:

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_3 = \frac{V_2}{7} = \frac{-14}{7} = -2A$



Find the Eff. Current through the 5Ω & 2Ω resistor

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0$ — ①

Taking KVL for LOOP 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ — ②

$7V_1 + 8V_2 = 63$ — ① ×1

$-V_1 + V_2 = 9$ ×7

$7V_1 + 8V_2 = 63$

$+ -7V_1 + 7V_2 = 63$

$15V_2 = 126$

From eqn 2: $-V_1 + V_2 = 9$

$V_2 = 9 + V_1$ Put into eqn 1

$7V_1 + 8[9 + V_1] = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = 63 - 72$

$15V_1 = -9$

$V_1 = -9/15$

$V_1 = -0.6V$

Sub $V_1 = -0.6$ in eqn 1

$-[-0.6] + V_2 = 9$

$0.6 + V_2 = 9$

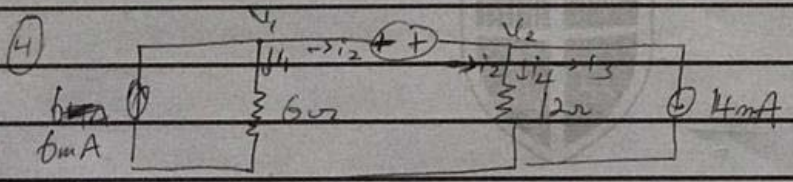
$V_2 = 9 - 0.6 = 8.4$

$V_1 = -0.6V \quad V_2 = 8.4V.$

Current through 3Ω and 2Ω.

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$ at → 3Ω

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$ at 2Ω



Find the nodes at the voltages and the currents through the 6Ω and 12Ω resistor.

Take $V_1 - V_2 = 6V \Rightarrow i_2$.

At node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + [V_1 - V_2]$

$36 = V_1 + 6[V_1 - V_2]$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad ①$

At Node 2.

$i_2 = i_3 + i_4$

$\frac{V_1 - V_2}{} = 4mA + \frac{V_2 - 0}{12}$

$12[V_1 - V_2] = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2$ - (2)

Solving V_1 and V_2 Simultaneously we have

$V_1 = 9.5V$

$V_2 = 5.1V$

Current through the 6Ω resistor:

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4A$

Current through 12Ω

$I_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V$ $V_2 = 5.1V$

$i_1 = 1.58V$ $I_4 = 0.48A$