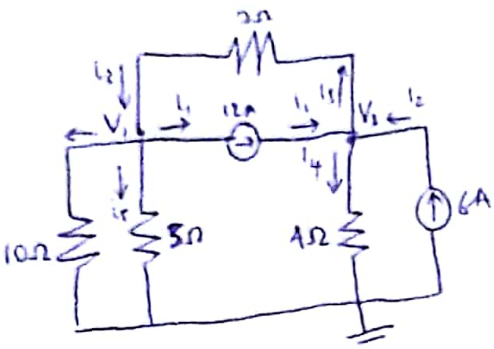


at node V_1

$$i_3 = i_1 + i_5 + i_6 = 12 + i_5 + i_6$$

$$\therefore\ 12 = i_3 - i_5 - i_6$$

$$12 = \frac{V_2 - V_1}{2} - \left[\frac{V_1 - 0}{5}\right] - \left[\frac{V_1 - 0}{10}\right]$$

$$12 = \frac{V_2 - V_1}{2} - \frac{V_1}{5} - \frac{V_1}{10}$$

$$12 = \frac{5(V_2 - V_1) - 2(V_1) - V_1}{10}$$

$$120 = 5V_2 - 5V_1 - 2V_1 - V_1$$

$$120 = 5V_2 - 8V_1 \quad —(1)$$

From node V_2

$$i_1 + i_2 = i_3 + i_4$$

$$12 + 6 = \frac{V_2 - V_1}{2} + \left[\frac{V_2}{4}\right]$$

$$18 = \frac{V_2 - V_1}{2} + \frac{V_2}{4}$$

$$18 = \frac{2(V_2 - V_1) + V_2}{4}$$

$$72 = 3V_2 - 2V_1 \quad —(2)$$

Using Simultaneous Eqn

$$120 = 5V_2 - 8V_1 \quad --- \times(2)$$

$$72 = 3V_2 - 2V_1 \quad --- \times(8)$$

$$\Rightarrow \quad 240 = 10V_2 - 16V_1$$

$$-\ 576 = 24V_2 - 16V_1$$

$$-336 = -14V_2$$

$$V_2 = 24_V$$

From Eqn (1)

$$120 = 5(24) - 8V_1$$

$$120 = 120 - 8V_1$$

$$8V_1 = 120 - 120$$

$$V_1 = 0V$$

Current flowing through the 2Ω resistor

$$= \frac{V_2 - V_1}{2} = \frac{24 - 0}{2} = 12A$$

while current flowing through the 4Ω resistor

$$= \frac{V_2}{4} = \frac{24}{4} = 36A$$

Due to V_1 being 0v then current moving through the 10Ω and 5Ω resistors both equals zero (0).

ii) V_1 and V_2 when 12A is replaced with 1A

at node V_1

$$1 = \frac{5(V_2 - V_1) - 2(V_1) - V_1}{10}$$

$$10 = 5V_2 - 8V_1 \quad —(1)$$

at node V_2

$$1 + 6 = \frac{V_2 - V_1}{2} + \frac{V_2}{4}$$

$$7 = \frac{2(V_2 - V_1) + V_2}{4}$$

$$28 = 3V_2 - 2V_1 \quad —(2)$$

Using Simultaneous Eqn

$$10 = 5V_2 - 8V_1 \quad — \times(2)$$

$$28 = 3V_2 - 2V_1 \quad — \times(8)$$

$$\Rightarrow \quad 20 = 10V_2 - 16V_1$$

$$-\ 224 = 24V_2 - 16V_1$$

$$-204 = -14V_2$$

$$\therefore V_2 = 14.57$$

and by extension using eqn (1)

$10 = 5(14.57) - 8V_1$

$10 = 72.86 - 8V_1$

$8V_1 = 62.86$

$V_1 = 7.86V$

Current through the 2Ω resistor

$= \frac{V_2 - V_1}{2} = \frac{14.57 - 7.86}{2}$

$= 3.355A$

Current through the 4Ω resistor

$= \frac{V_2}{4} = \frac{14.57}{4} = \cancel{36}$ 3.6425A

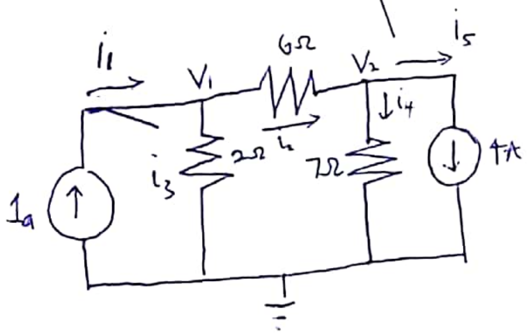
Current through the 10Ω resistor

$= \frac{V_1}{10} = \frac{7.86}{10} = 0.786A$

Current through the 5Ω resistor

$= \frac{V_1}{5} = \frac{7.86}{5} = 1.572A$

b)



at node 1

$i_1 = i_2 + i_3$

$1 = \left[\frac{V_1 - V_2}{6}\right] + \left[\frac{V_1 - 0}{2}\right]$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$1 = \frac{V_1 - V_2 + 3V_1}{6}$

$6 = 4V_1 - V_2 \quad —(1)$

at node 2

$i_2 = i_4 + i_5$

$\left[\frac{V_1 - V_2}{6}\right] = \left[\frac{V_2}{7}\right] + 4$

$\frac{V_1 - V_2}{6} - \frac{V_2}{7} = 4$

$\frac{7(V_1 - V_2) - 6V_2}{42} = 4$

$168 = 7V_1 - 13V_2 \quad —(2)$

Solving simultaneously

$V_1 = -2V$ and $V_2 = -14V$

From this current moving through the 6Ω

$= \cancel{4\Omega}\ \frac{V_1 - V_2}{6}$

$= \frac{-2-14}{6} = \frac{-16}{6}$

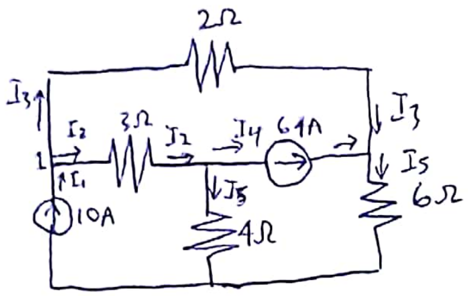
$= -2.67A$

From this current moving through the 2Ω resistor

$= \frac{V_1}{2} = \frac{-2}{2} = 1A$

Current moving through 7Ω

$= \frac{V_2}{7} = \frac{-14}{7} = -2A$



From node 1

$$I_1 = I_2 + I_3$$

$$\Rightarrow 10 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2}$$

$$10 = \frac{(V_1 - V_2)2 + (V_1 - V_3)3}{6}$$

$$60 = 2V_1 - 2V_2 + 3V_1 - 3V_3$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{—(1)}$$

From node 2

$$I_2 = I_5 + I_4$$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{4} + 64$$

$$\frac{V_1 - V_2}{3} - \frac{V_2}{4} = 64$$

$$\frac{4V_1 - 4V_2 - 3V_2}{12} = 64$$

$$\frac{4V_1 - 7V_2}{12} = 64$$

$$4V_1 - 7V_2 = 768 \quad \text{—(2)}$$

From node 3

$$I_4 + I_3 = I_5$$

$$\cancel{64 + \frac{V_2}{4} = \frac{V_3}{6}}$$

$$\cancel{64 = \frac{V_3}{6} - \frac{V_2}{4}}$$

$$\cancel{\frac{2V_3 - 3V_2}{12} = 64}$$

$$\cancel{768 = 2V_3 - 3V_2}$$

$$64 + \frac{V_1 - V_3}{2} = \frac{V_3}{6}$$

$$64 = \frac{V_3}{6} - \frac{(V_1 - V_3)}{2}$$

$$64 = \frac{V_3 - 3V_1 + 3V_3}{6}$$

$$\cancel{64 = \frac{(6V_1 + 4V_3)}{6}}$$

$$\cancel{768 = 6V_1 - 4V_3}$$

$$384 = 4V_3 - 3V_1 \quad \text{—(3)}$$

$\therefore$

$$60 = 5V_1 - 2V_2 - 3V_3$$
$$768 = 4V_1 - 7V_2 + 0V_3$$
$$384 = -3V_1 + 0V_2 + 4V_3$$

Using Cramer's Rule

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}; \quad V_2 = \frac{\Delta_2}{\Delta}; \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$$= 5\begin{vmatrix} -7 & 0 \\ 0 & 4 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ -3 & 4 \end{vmatrix} - 3\begin{vmatrix} 4 & -7 \\ -3 & 0 \end{vmatrix}$$

$$= 5(-28 - 0) + 2(16 - 0) - 3(0 - 21)$$

$$= -140 + 32 + 63 = -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$$= 60(-28) + 2(3072-0) - 3(0+2688)$$

$$= -1680 + 6144 - 8064$$

$$= 6144 - 9744$$

$$= -3600$$

$$\therefore V_1 = \frac{-3600}{-45} = 80\,V$$

$$\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$$= 5(3072-0) - 60(16-0) - 3(1536+2304)$$

$$= 15360 - 960 - 11520$$

$$= 15360 - 12480$$

$$= 2880$$

$$\therefore V_2 = \frac{2880}{-45} = -64\,V$$

$$\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5(-2688-0) + 2(1536+2304) + 60(0-21)$$

$$= -13440 + 7680 - 1260 = -7020$$

$$\therefore V_3 = \frac{-7020}{-45} = 156\,V$$

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17/ENG05/013

MECHATRONICS ENGINEERING