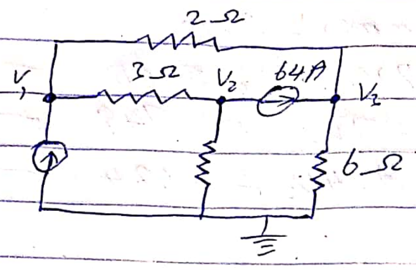
Elenwoke Chukwuebmeka Joshua
17/ENG04/020
Electrical/Electronics

1.) Find the Voltages at nodes, 1, 2 and 3 in the circuit below.



At node 1, KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 + V_3}{2} + \frac{V_1 + V_2}{3}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 + 3V_3$ — (i)

At node 2, KCL

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2 - 0$

$768 = 4V_1 - 7V_2$ — (ii)

At node 3, KCL

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

Scanned with CamScanner

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ — (iii)

Using Crammer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ — (i)

$4V_1 - 7V_2 = 748$ — (ii)

$-3V_1 + 4V_3 = 384$ — (iii)

In Matrix Representation

$$\begin{pmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 60 \\ 748 \\ 384 \end{pmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$, $V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$= 60(-28-0) - 768(-8-0) - 384(0)$

$= -1680 + 6144 - 8064$

$\Delta_1 = -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5(768-0) - 4(240 + 1152) - 3(0 + 2244)$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$V_3 = \Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

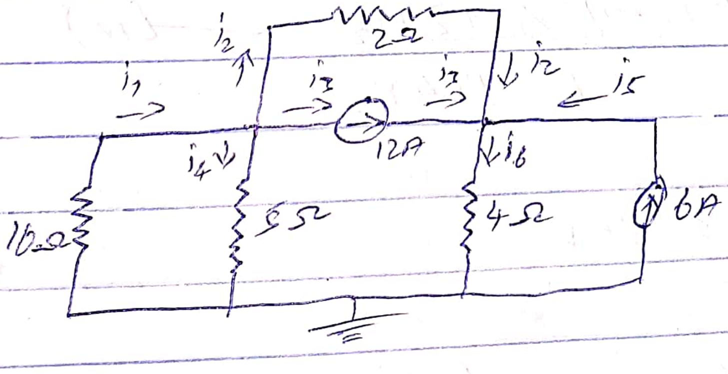
$= -5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) -$
$- 3((-2 \times 768) - (-7 \times 60))$

$= -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

$\therefore V_1 = 80V,\ V_2 = -64V,\ V_3 = 156V$

2. Find the voltage at nodes 1 and 2 and determine the current flowing through the four resistance in the circuit below



At node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ — (i)

At node 2, KCL

$i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ — (ii)

Using Elimination Method

$120 = -8V_1 + 5V_2 \quad \times 2$

$144 = -4V_1 + 6V_2 \quad \times 4$

$240 = -16V_1 + 10V_2$ — (iii)

$576 = -16V_1 + 24V_2$ — (iv)

$816 \quad 0 + 34V_2$

$V_2 = 24V$

Sub V_2 in eqn (iii)

$240 = -16V_1 + 240$

$V_1 = 0$

$\therefore V_1 = 0V, V_2 = 24V$

$i_1 = 0A; i_2 = 0A; i_3 = 6A; i_4 = -12A$

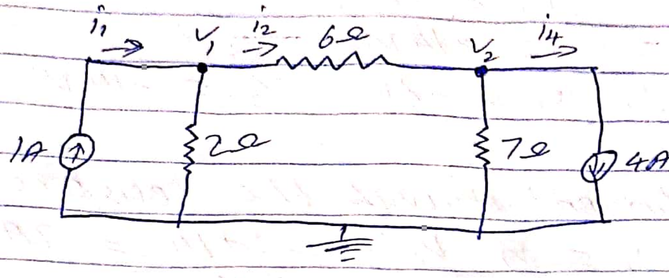
2ii: Obtain V_1 ...

resistors ...

2A cur...

2A.

2ii. Obtain V_1 and V_2 and the current through the resistors for the current in example (ii) at the 2A current source was replaced by a 1A current 2A.



Solution

At node 1, KCL

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = (V_1 - V_2) + 3V_1$$

$$6 = 4V_1 - V_2 \quad \text{— (i)}$$

At node 2, KCL

$$i_2 = i_4 + i_5$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \quad \text{— (ii)}$$

From equ (i), $V_2 = 4V_1 - 6$

Sub V_2 in equ (ii)

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$168 - 78 = 7V_1 - 52V_1$$

$$90 = -45V_1$$

$$V_1 = -2V$$

Scanned with CamScanner

Sub $V_1 = -2$ into eqn (1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$= -14V$

$\therefore V_1 = -2V, \quad V_2 = -14V$

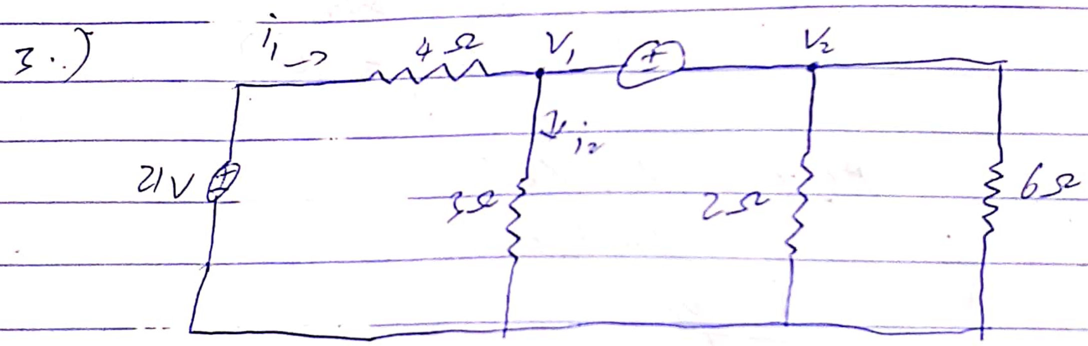
Current through the resistors

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

3.)



Find the current through the 3Ω and 2Ω resistors

Solution

Using KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0 \quad (1)$

Using KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ — (ii)

$7V_1 + 8V_2 = 63$ — ×1

$-V_1 + V_2 = 9$ — ×7

$7V_1 + 8V_2 = 63$ — (iii)

$-7V_1 + 7V_2 = 63$ — (iv)

$0 + 15V_2 = 126$

$V_2 = \frac{126}{15} = 8.4V.$

Sub V_2 into equ (iv)

$-7V_1 + 7(8.4) = 63$

$-7V_1 = 63 - 58.8$

$-7V_1 = 4.2$

$V_1 = -0.6V$

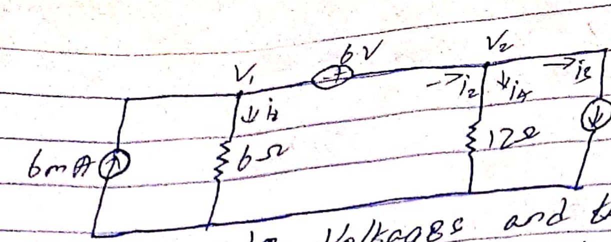
Current through the 2Ω resistor

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4.)



Find the node Voltages and the current through the 6Ω and 12Ω resistor

solution

let say $V_1 - V_2 = 6V \rightarrow i_2$

At node 1, using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$0.036 = V_1 + 6(V_1 - V_2)$$

$$0.036 = V_1 + 6V_1 - 6V_2$$

$$0.036 = 7V_1 - 6V_1 \quad -(i)$$

At node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 0.048 + V_2$$

$$0.048 = 12V_1 - 12V_2 - V_2$$

$$0.048 = 12V_1 - 13V_2 \quad -(ii)$$

$$7V_1 - 6V_2 = 0.036 \times 12$$

$$12V_1 - 13V_2 = 0.048 \times 7$$

$$84V_1 - 72V_2 = 0.432 \quad -(iii)$$

$$84V_1 - 91V_2 = 0.336 \quad -(iv)$$

$$0 + 19V_2 = 0.096$$

$V_2 = 5.053 \times 10^{-3} V$

sub V_2 in equ (iv)

$84V_1 - 0.4598 = 0.336$

$84V_1 = 0.336 + 0.4598$

$V_1 = \frac{0.7958}{84} = 9.474 \times 10^{-3} V$

$\therefore$ current through the 6 Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.474 \times 10^{-3}}{6} = 1.579 \times 10^{-3} A$

current through the 12 Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.053 \times 10^{-3}}{12} = 4.212 \times 10^{-4} A$

$V_1 = 9.474 \times 10^{-3} V$

$V_2 = 5.053 \times 10^{-3} V$

$i_1 = 1.579 \times 10^{-3} A$

$i_4 = 4.212 \times 10^{-4} A$