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~~18~~ENG04/079
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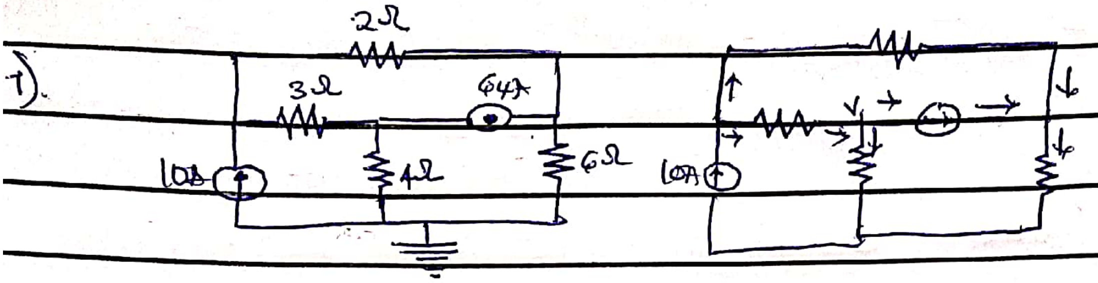
EEE 322

1).



At node 1, KCL

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \text{ ----- } (1)$$

At Node 2, KCL

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \text{ ---- } (ii)$$

At Node 3, KCL

$$64 + i_4 = i_5$$

$$64 = i_5 - i_4$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_2}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$$384 = -3V_1 + 4V_3 \text{ ------ } (iii)$$

Using Gramer's Rules

$5V_1 - 2V_2 - 3V_3 = 60$ ——— (i)

$4V_1 - 7V_2 = 768$ ——— (ii)

$-3V_1 + 4V_3 = 384$ ——— (iii)

In matrix representation

$$\begin{bmatrix} 3 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 765 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$ $\quad V_2 = \frac{\Delta_2}{\Delta}$ $\quad V_3 = \frac{\Delta_3}{\Delta}$

Where $\Delta_1 \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$ $+ - +$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For V_2 : Δ_2 $\begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5[(768 \times 4) - 0] - 4[(60 \times 4) - (384 - 3)] - 3[0 - (768$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

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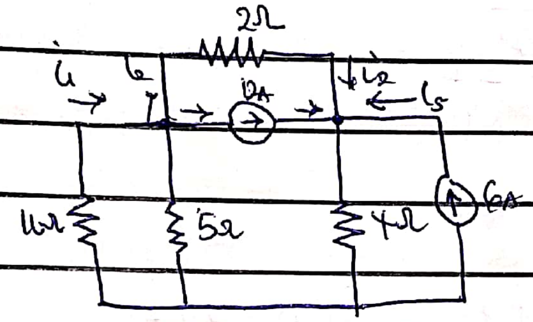
For V_3 ;
$$\begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5[(-7\times384)-0] - 4[(-2\times384)-0] - 3[(-2\times768)-(7\times60)]$

$\therefore V_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{7020}{-45} = 156V$

Hence V_2 − for $V_2 = -64V$, $V_3 = 156V$

2)



~~At~~ Node 1 ; KCL

$i_1 = i_2 + i_3 + i_4$

$\dfrac{V_0 - V_1}{10} = \dfrac{V_1 - V_2}{2} + 12 + \dfrac{V_1 - V_0}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V + 5V_2$ ------- (i)

~~At~~ Node 2

$i_3 + i_2 + i_5 = i_6$

$12 + \dfrac{V_1 - V_2}{2} + 6 = \dfrac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ ------ (ii)

Using Elimination method.

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$120 = -8V_1 + 5V_2$ ——— (i) $\times -4$

$144 = -4V_1 + 6V_2$ ——— (ii) $\times -8$

$\times 60)$ $-480 = 32V_1 - 20V_2$ ——— (iii)

$-1152 = 32V_1 - 28V_2$ ——— (iv)

Subtract eqn (iii) from

$-672 = 0 - 28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24V$

Subs $V_2 = 24$ in eqn ii

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6V_2}{-4}$

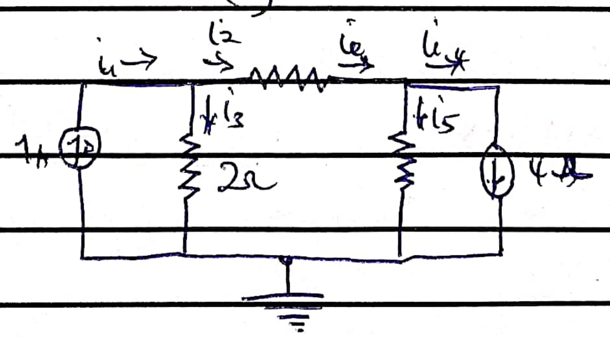
$V_1 = \frac{144 - 6V_2}{-4}$

$V_1 = 0$

$\therefore V_1 = 0V \cdot V_2 = 24V$

$i_1 = 0A$ $i_2 = 0A$ $i_3 = 6A$ $i_4 = 12A$

(iii)



At Node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + V_1/2$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ ------ (i)

At Node 2

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ ------ (ii)

from eqn (i); $V_2 = 4V_1 - 6$

Sub $V_2 = 4V_1 - 6$ in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Sub $V_1 = -2$ in eqn (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

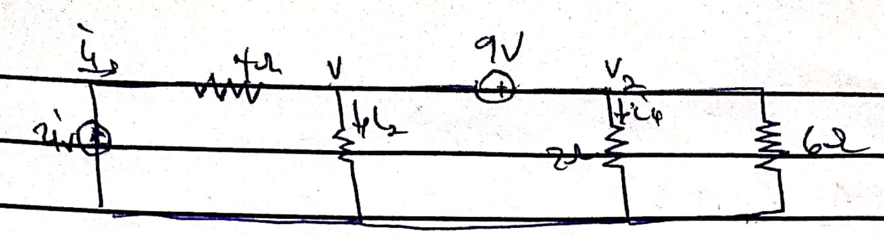
$\therefore V_1 = -2V, \; V_2 = -14V$

Current through the resistors

$$i_2 = \frac{V_1 - V_2}{6} = \frac{2 + 14}{6} =$$

$$i_3 = V_1/2 = \frac{-2}{2} = -1A$$

$$i_3 = V_2/7 = \frac{-14}{7} = -2A$$

3) 

find the current through the 3Ω and 2Ω resistor

Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 2i}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \text{---- (ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{—— (i)}$$

$$-V_1 + V_2 = 9 \quad \text{—— (ii)}$$

let $\quad V_2 = 9 + V_1 \quad$ —— (iii)

Sub. eqn (iii) in (i)

$$7V_1 = 8(9+V_1) = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = -9$$

$$V_1 = 0.6V$$

Sub $V_1 = -0.6$ in eqn (ii)

$$-(-0.6) + V_2 = 9$$

$$0.6 + V_2 = 9$$

$$V_2 = 8.4V$$

$$\therefore V_1 = -0.6V \text{ and } V_2 = 8.4V$$

Current through the 3Ω resistor

$$i_2 = V_1/3 = -0.6/3 = 0.2A$$

Current through the 2Ω resistor

$i_4 = \frac{V_2}{4} = \frac{8.4}{6r4} = 4.2A$

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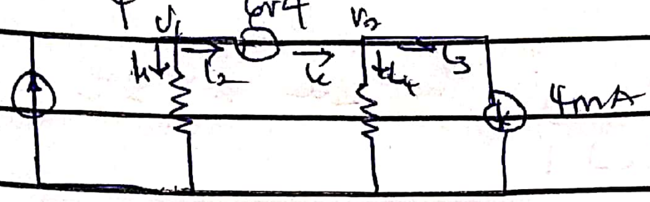

(a) Node 1 using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 + V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_2 + 6V_2$ —— (i)

(2) Node (ii)

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 - 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2$ —— (ii)

Solving V_1 and V_2 simultaneously we have

$V_1 = 9.5V$ and $V_2 = 5.1V$

∴ Current through 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$; $i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4A$

Current through 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

∴ $V_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_4 = 0.43A$