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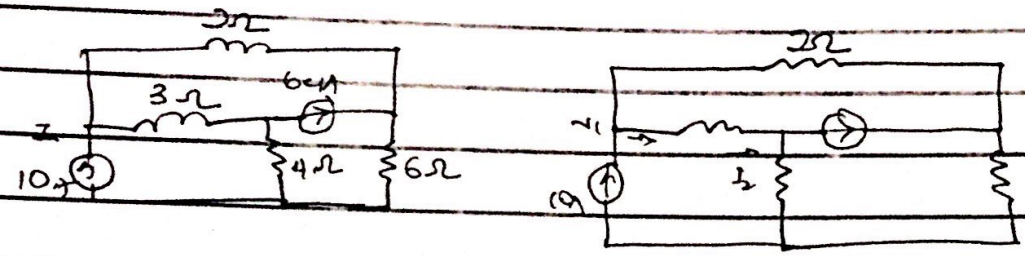
18/ENG04/078
Elect/Elect

(1) Find the voltage at node 1, 2 & 3 in the circuit below



Node 1 KCL

$$10 = i_1 + i_2 \rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

$$60 = 3[V_1 - V_3] + 2[V_1 - V_3]$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 3V_3 - 2V_2 \quad -①$$

At Node 2 KCL

$$I_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4[V_1 - V_2] - 3[V_2 - 0]$$

$$768 = 4V_1 - 7V_2 \quad -②$$

Node 3 KCL

$$64 + I_1 = 15$$

$$64 = i_5 - i_4$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$384 = V_3 - 3[V_1 - V_2]$

$384 = -3V_1 + 4V_3 \quad -c$

$5V_1 - 2V_2 - 3V_3 = 60 \quad -a$

$4V_1 - 7V_2 + 10V_3 = 768 \quad -b$

$-3V_1 + 0V_2 + 4V_3 = 384$

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta} \quad V_2 = \frac{\Delta_2}{\Delta} \quad V_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5[-28-0] + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60[-28-0] - 768[-8-0] - 384[0-21]$

$= 7680 + 6144 - 8064$

$= -3600$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = \frac{80V}{2}$$

$$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -0 & 384 & 4 \end{vmatrix}$$

$$= 5[(708 \times 4) - 0] - 4[60 \times 4) - [384 - 3)] - 3[0 - (708 - 3)]$$

$$= -2880$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

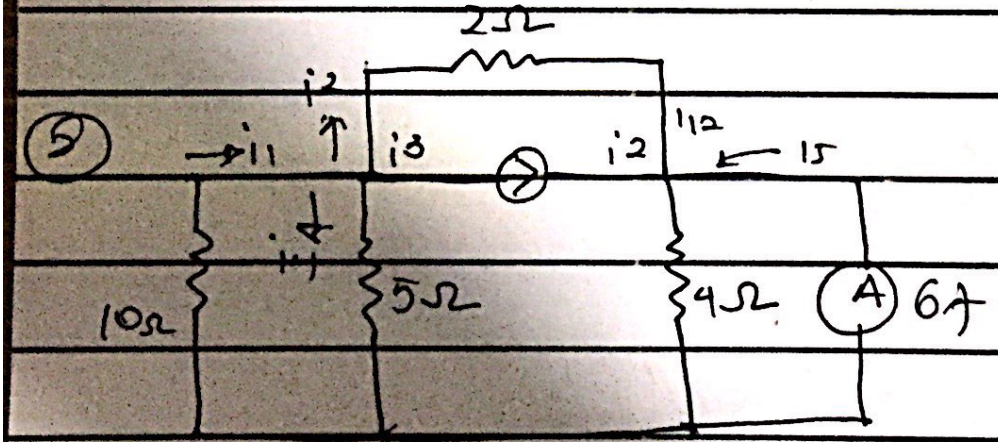
$$V_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 768) - [-7 \times 60]$$

$$= -7020$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{45} = 156$$

$V_1 = 80V$ $V_2 = 64$ $V_0 = 156V$



Node KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_3}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5[V_1 - V_2] + 120 + 2[V_1 - 0]$

$-V_1 = 5V_1 - 5V_2 + 120 - 2V_1$

$120 = -8V_1 + 5V_2$ — ①

Node 2

$i_3 + i_4 + i_5 = 16$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4[V_1 - V_2] + 48 = 2[V_2]$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ — ②

Using elimination method

$120 = -8V_1 + 5V_2$ — ① $\times 4$

$144 = -4V + 6V_2$ — ② $\times 8$

$-480 = 32V_1 - 20V_2$ — ③

$-1152 = 32V_1 - 48V_2$ — ④

Subtract eqn 3 from 4

$-672 - 0 = -28V_2$

$-672 = -28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24V$

Sub V_2 into eqn 2

$144 = -4V_1 + 6V_2$

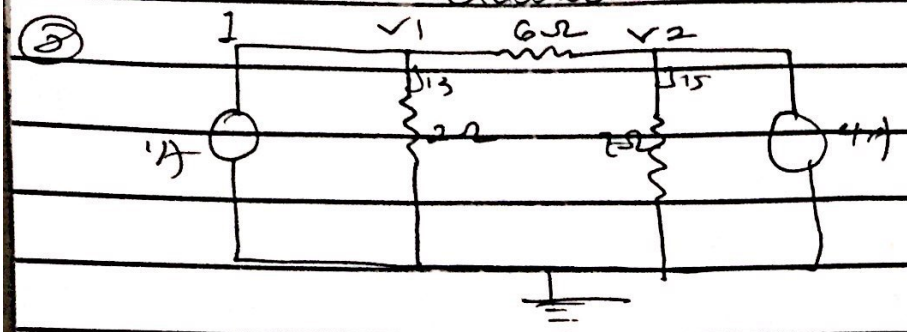
$V_1 = \frac{144 - 6[24]}{-4}$

$$V_1 = \frac{144-144}{-4} = 0V$$

$V_1 = 0V \quad V_2 = 24V$

~~$i_1 = 0A, i_2 = 0A, i_3 = 6A, i_4 = -10A$~~

Classmate



Node 1

$I_0 = i_1 = i_2 + i_3$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \quad (1)$$

Node (2)

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} - 4 + \frac{V_2}{7}$$

$$7[V_1 - V_2] = 16 + 6V_2$$

$$16 = 7V_1 - 13V_2 \quad (2)$$

Cramer (1)

$V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ into eqn 2

$168 = 7V_1 - 15[4V_1 - 6]$

$168 = 7V_1 - 22V_1 + 78$

$90 = 45V_1$

$V_1 = 90/45$

$V_1 = -2V$

Subs $V_1 = -2$ into eqn 1

$6 = 4[-2] - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

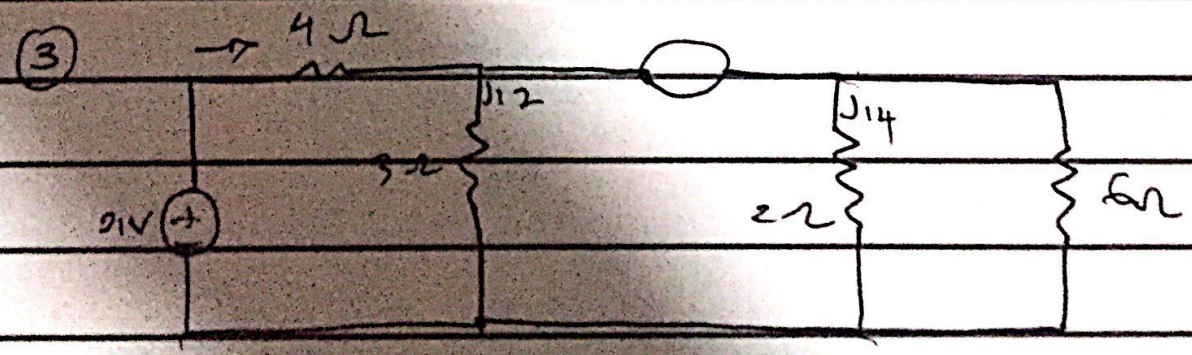
$V_1 = -2V$ & $V_2 = -14V$

Current through the resistance

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$



Find the current through 3Ω & 2Ω 1Ω

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{v_1 - 21}{4} + \frac{v_1}{3} + \frac{v_2}{6} + \frac{v_3}{2}$

$7v_1 + 8v_2 - 63 = 0$ — (i)

taking KVL for loop 1

$-v_1 - 9 + v_2 = 0$

$-v_1 + v_2 = 9$ — (ii)

$7v_1 + 8v = 63$ (i) $\times 1$

$-v_1 + v_2 = 9$ $\times 7$

$7v_1 + 8v_2 = 63$

$-7v_1 + 7v = 63$

$11v = 10.6$

from eqn 2 $-v_1 + v_2 = 9$

$v_2 = 9 + v_1$ put into eqn (i)

$7v_1 + 8(9 + v_1) = 63$

$7v_1 + 72 + 8v_1 = 63$

$15v_1 = 63 - 72$

$15v_1 = -9$

$v_1 = -9/15$

$v_1 = -0.6V$

sub $v_1 = 0.6$ In eqn (ii)

$-[-0.6]+V_2 = 9$

$0.6+V_2 = 9$

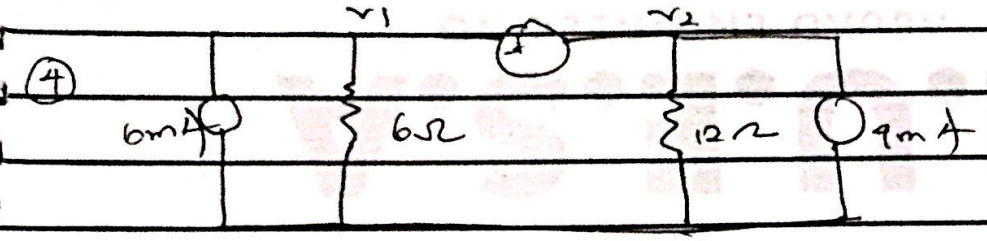
$V_2 = 9 - 0.6 = 8.4\,V$

$V_1 = 0.6$ or $V_2 = 8.4\,V$

Current through 3Ω and 2Ω

$i_2 = \frac{V}{3} = \frac{-0.6}{3} = -0.2\,A$ at 3Ω

$i_4 = \frac{V}{4} = \frac{8.4}{4} = 4.2\,A$ at 4Ω



Find the node voltage and the current through 6Ω & 12Ω resistor

Take $V_1 - V_2 = 6V$ → i_2

At node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + [V_1 - V_2]$

$36 = V_1 + 6[V_1 - V_2]$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ ①

At node 2

$i_2 = i_3 + i_4$

$V_1 = V_2 - 4mA + \frac{V_2 - 0}{12}$

$12[V_1 - V_2] = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2$ — ③

Solving simultaneously

$V_1 = 9.5V$

$V_2 = 5.1V$

Current through the 6Ω resistor

$i_1 = V_1/6 = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4$

Current through I_2

$I_4 = \frac{V_1}{I_2} = \frac{5.1}{I_2} = 0.43A$

$V_1 = 9.5V$ $I_2 = 5.1V$

$i_1 = 1.58V$ $I_4 = 0.43A$