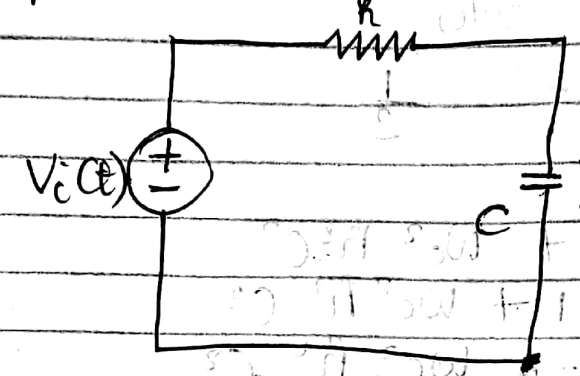
UDEMADU CELESTINE CHIBUIKE
17/ENG05/039
Mechatronics Engr.
Laplace - ENG 322 Assignment.



Determine the type of filter shown ~~where~~ above and show that its cut-off frequency is $\omega_c = 1/RC$.

SOLN.

Converting to frequency domain.

$R = R$

$C = 1/j\omega C$

$H(\omega) = \frac{V_o}{V_i}$

Using KVL (Series connection)

$H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1/j\omega}{\frac{Rj\omega C + 1}{j\omega}}$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

where $H(0) = 1$, $H(\infty) = 0$

The half power frequency which is equivalent to the corner frequency on the BODE plots but in the context of filters is called CUT-OFF FREQUENCY ω_c, is obtained by setting magnitude of $H(\omega)$ $= \frac{1}{\sqrt{2}}$

Substituting ω_c

$$|H(\omega_c)| = \frac{1}{\sqrt{1+\omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

Square both sides

$$\frac{1^2}{1+\omega_c^2 R^2 C^2} = \frac{1}{2}$$

$$2 \cdot 1^2 = 1 + \omega_c^2 R^2 C^2$$

$$2 = 1 + \omega_c^2 R^2 C^2$$

$$2 - 1 = \omega_c^2 R^2 C^2$$

$$1 = \omega_c^2 R^2 C^2$$

Taking Square root of both sides

$$\sqrt{1} = \sqrt{\omega_c^2 R^2 C^2}$$

$$1 = \omega_c RC$$

$$\therefore \omega_c = \frac{1}{RC}$$

$$\omega_c = \frac{1}{\frac{RC}{2}}$$