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$$216 = \frac{2 \times 6}{2+6} = \frac{12}{8} = 1.5$$

find the current through 3 Ω resistor
using KCL at node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

Node 1

KCL

$$V_1 - 20 + \frac{V_1}{3} + \frac{V_1}{6} + \frac{V_1}{2} = 0$$

$$\Rightarrow 3(V_1 - 20) + 2V_1 + 3V_1 + 6V_1 = 0$$

$$= 3V_1 - 60 + 2V_1 + 3V_1 + 6V_1 = 0$$

$$= 7V_1 - 60 = 0$$

$$7V_1 = 60 \Rightarrow V_1 = 8.57 \text{ V}$$

using KCL at node 2

$$\Rightarrow -4 - 9 + V_2 = 0$$

$$-4 + V_2 = 9 \Rightarrow V_2 = 13 \text{ V}$$

$$7V_1 + 9V_2 = 60$$

$$7 \times 8.57 + 9 \times 13 = 60$$

using my calculator to solve simultaneously

$$V_1 = 8.57 \text{ V}, V_2 = 13 \text{ V}$$

• The current through 3 Ω resistor

using Ohm's law

$$I_{3\Omega} = \frac{V_1}{3}$$



find the current through 3Ω



$$= \frac{0.6 - 0}{5} = \frac{-0.6}{5}$$

$$\therefore I_{AB} = -0.12 \text{ A}$$

∴ The Current through the 2Ω is 0.12A

$$I_{AB} = \frac{V_{AB}}{R}$$

$$= \frac{0.6 - 0}{5} = \frac{0.6}{5}$$

$$I_{AB} = 0.12 \text{ A}$$

(4)



Find the voltages and currents through 6Ω.

12V



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$$\rightarrow V_1 - V_2 = 6V$$

$$\text{Assume } V_1 - V_2 = 6V$$

Using KVL at node 1

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1}{6} + (V_1 - V_2)$$

$$6mA = \frac{V_1 + 6(V_1 - V_2)}{6}$$

$$6mA = \frac{V_1 + 6V_1 - 6V_2}{6}$$

$$36mA = 7V_1 - 6V_2$$

$$\text{At node 2; } i_2 = i_4 + i_3 \quad i_3 = 4mA$$

$$\rightarrow i_2 = i_4 + 4mA \quad \therefore 4mA = i_2 - i_4$$

$$4mA = (V_1 - V_2) - \frac{V_2}{12}$$

$$4mA = \frac{12(V_1 - V_2) - V_2}{12}$$

$$48mA = 12V_1 - 13V_2 - V_2$$

$$48mA = 12V_1 - 13V_2 \quad \text{--- (2)}$$

Solving (1) and (2)

$$V_1 = 12V \quad V_2 = 6V$$

Current through the 6Ω resistor

$$i = \frac{V_1}{6} = \frac{12V}{6} = 2A$$

$$10 = \frac{5}{2}V_1 - \frac{1}{2}V_2 - \frac{1}{2}V_3$$

$$6V_1 = \frac{1}{2}V_2 - \frac{1}{2}V_3 + 0V_4$$

$$6V_1 = -\frac{1}{2}V_2 - 0V_3 + \frac{1}{2}V_4$$

Using matrix method

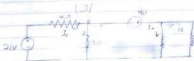
$$\begin{bmatrix} 10 \\ 6V_1 \\ 6V_1 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$C \mathbf{V} = B \mathbf{A}$$

$$B \mathbf{A} = \begin{bmatrix} 1.5 & -0.5 & -0.5 \\ 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 10 \\ 6V_1 \\ 6V_1 \end{bmatrix}$$

$$C = \begin{bmatrix} 20 \\ -64 \\ -56 \end{bmatrix}$$

$$\therefore V_1 = 90V, V_2 = -60V, V_3 = 150V$$



$$\Rightarrow \frac{V_1 - V_2}{2} - \frac{V_2}{4} = -18$$

$$\frac{2(V_1 - V_2)}{4} - \frac{V_2}{4} = -18$$

$$\frac{2V_1 - 2V_2 - V_2}{4} = -18$$

$$\frac{2V_1 - 3V_2}{4} = -18$$

$$2V_1 - 3V_2 = -72 \quad \dots \dots (11)$$

$$5V_1 - 5V_2 = 120$$

$$2V_1 - 3V_2 = -72$$

Using Calculator to solve simultaneously

$$V_1 = 0V \quad \& \quad V_2 = 24V$$

$$i_1 = \frac{V_1}{10} = \underline{0A}$$

$$i_2 = \frac{V_2}{5} = \underline{0A}$$

$$i_3 = \frac{V_2}{4} = \underline{6A}$$

$$i_4 = \frac{4 - V_2}{2} = \frac{0 - 24}{2} = \underline{-12A}$$

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Find the voltages of nodes 1, 2 & 3



at node 1, using KCL: $10 = i_1 + i_2$
 $10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

at node 2, using KCL: $-i_1 = 6i_3 + i_4$
 $\frac{V_2 - V_1}{2} = 6i_3 + \frac{V_2 - V_3}{4}$

at node 3, using KCL: $i_4 = 6i_3 + i_5$
 $\frac{V_3}{6} = 6i_3 + \frac{V_3 - V_2}{4}$

$10 = \frac{V_1}{2}$
 $6i_3 = i_4$
 $6i_3 = i_5$

Using matrix
 $\begin{bmatrix} 10 \\ 6i_3 \\ 6i_3 \end{bmatrix} =$

$G \mathbf{V} =$
 $\mathbf{B}^T \mathbf{I} =$

$G =$

$\mathbf{B}^T \mathbf{I} =$



2

from node 1; using KCL

$$i_1 + i_2 + i_3 = -12A$$

$$i_1 = \frac{V_1 - 0}{10} = \frac{V_1}{10}$$

$$i_2 = \frac{V_1 - 0}{5} = \frac{V_1}{5}$$

$$i_3 = \frac{V_1 - V_2}{2}$$

$$\frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -12$$

$$\Rightarrow \frac{V_1}{10} + \frac{2V_1}{10} + \frac{5(V_1 - V_2)}{10} = -12$$

$$\Rightarrow \frac{V_1 + 2V_1 + 5V_1 - 5V_2}{10} = -12$$

$$\frac{8V_1 - 5V_2}{10} = -12$$

$$\Rightarrow 8V_1 - 5V_2 = -120$$

Applying KCL at node 2

$$i_4 + i_5 = i_3$$

$$i_4 - i_5 = -i_3$$

$$i_4 = \frac{V_2 - 0}{10} \quad i_5 = \frac{V_2 - 0}{10} \Rightarrow \frac{V_2}{10} - \frac{V_2}{10} = -i_3$$