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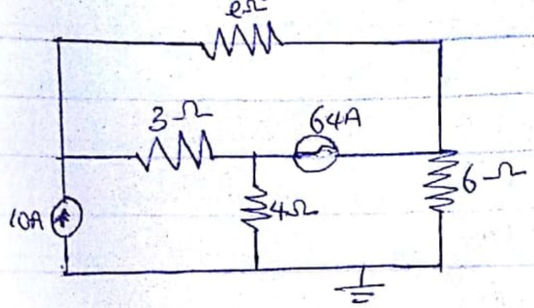
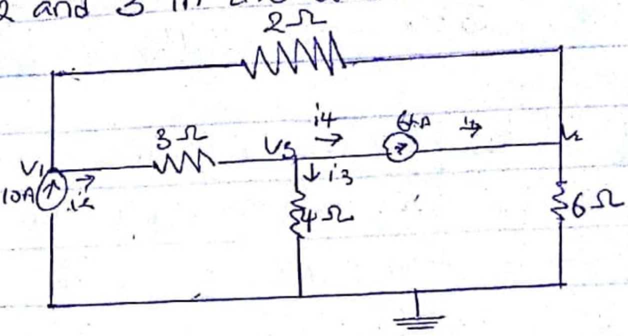
17/ENG02/066

1) Find the voltages at nodes 1, 2 and 3 in the circuit below





At node 1, KCL;

$10 = i_1 + i_2$

$10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_2 \ldots (i)$

At node 2, KCL;

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{V_3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2 + 0$

$768 = 4V_1 - 7V_2 \ldots (ii)$

At node 3, KCL

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

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$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$

Using Cramer's Rule

$5V_1 - 2V_2 - 3V_3 = 60 \cdots (i)$

$4V_1 - 7V_2 = 768 \cdots (ii)$

$-3V_1 + 4V_3 = 384 \cdots (iii)$

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$

$$\text{Where } \Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60(-28-0) - 768(-8-0) + 384(0-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For V_2: $\Delta = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

~~$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - 768 \ldots)$~~

~~$= 2880$~~

~~$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64$~~

$= 5((768 \times 4) - (384 \times 0)) - 60((4 \times 4) - (-3 \times 0)) + 3((4 \times 384) - (-3 \times 768))$

$= 5(3072 - 0) - 60(16 - 0) - 3(1536 + 2304)$

$= 5(3072) - 60(16) - 3(3840)$

$= 15360 - 960 - 11520$

$= 2880V$

For V_3; $\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

$= 5((-7 \times 384) - 0) - 4((-2 \times 384 - 0) - 3((-2 \times 768) - (-7 \times 60))$

$= -7020$

$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

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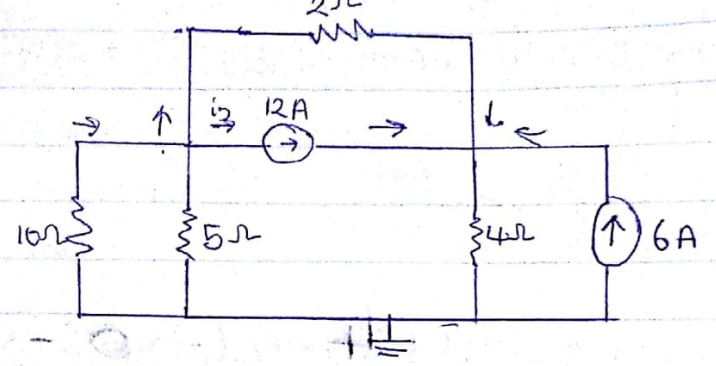
2) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below



At node 1; KCL

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \ldots (i)$

At node 2

$i_3 + i_2 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \ldots (ii)$

Using elimination method

$\times 4 \quad 120 = -8V_1 + 5V_2 \quad \ldots (i)$

$\times -8 \quad 144 = -4V_1 + 6V_2 \quad \ldots (ii)$

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$-480 = 32v_1 - 20v_2$ —— iii

$-1152 = 32V_1 - 48V_2$ —— iv

Subtract eqn (iii) from (iv)

$-672 = 0 - 28V_2$

$$V_2 = \frac{-672}{-28}$$

$V_2 = 24v$

Subs $V_2 = 24$ in eqn ii)

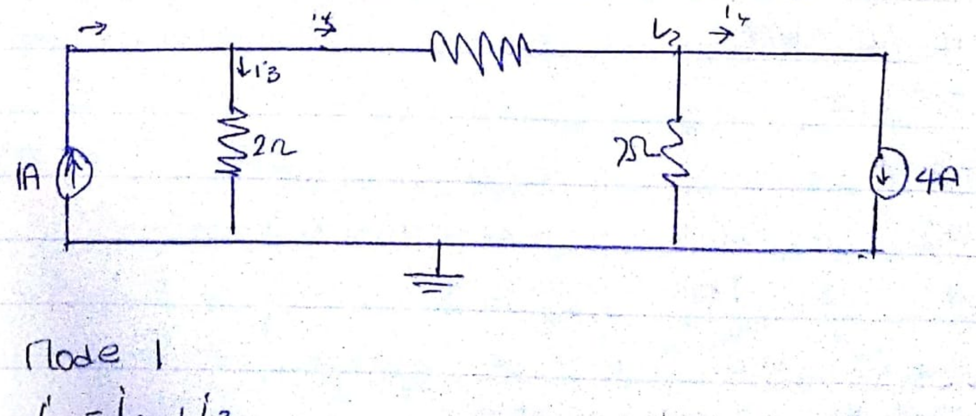
$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}$$

$V_1 = 0$

$\therefore V_1 = 0v, V_2 = 24v$

4) Obtain V_1 and V_2 and the currents through the resistor for the circuit in example (ii) if the 2A current source was replaced by a 1A current source



At node 1

$i = i_2 + i_3$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ ——— (i)

At node 2

$i_2 = i_4 + i_3$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ ——— (ii)

from eqn (i), $V_2 = 4V_1 - 6$

subs $V_2 = 4V_1 - 6$ in equa (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = \frac{90}{-45}$

$V_1 = -2V$

Sub. $V_1 = -2$ in equ (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$V_1 = -2V$, $V_2 = -14V$

Current through the resistor

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

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