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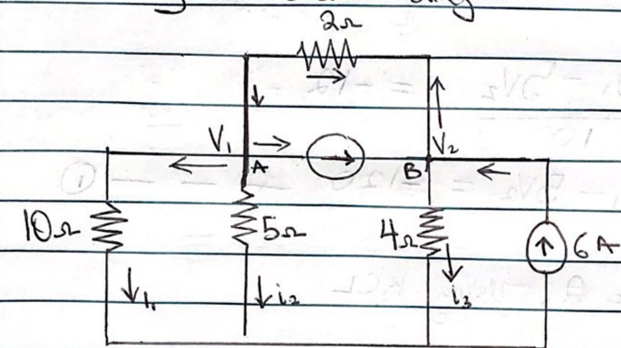
17/ENG04/048
Electrical/Electronics Engineering
EEE 322 Circuit Theory 2 Assignment

Solution

(1) Redrawing the circuit diagram



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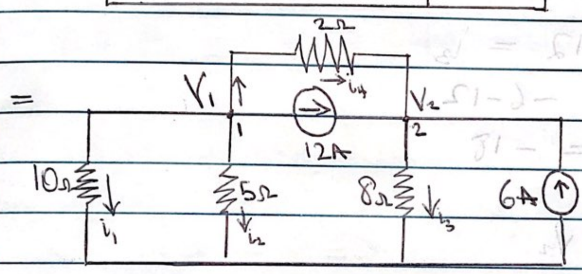


(2) From node 1, using KCL

$i_1 + i_2 + i_4 = -12A$

$(\Rightarrow i_1 + i_2 + i_4 + 12 = 0)$

$\Rightarrow i_1 = \frac{V_1 - 0}{10} = \frac{V_1}{10}$

$i_2 = \frac{V_1 - 0}{5} = \frac{V_1}{5}$

$i_4 = \frac{V_1 - V_2}{2}$

$$\Rightarrow \frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -12$$

$$= \frac{V_1}{10} + \frac{2V_1}{10} + \frac{5(V_1 - V_2)}{10} = -12$$

$$\Rightarrow \frac{V_1 + 2V_1 + 5V_1 - 5V_2}{10} = -12$$

$$\Rightarrow \frac{8V_1 - 5V_2}{10} = -12$$

$$\Rightarrow 8V_1 - 5V_2 = -120 \quad --- \text{(i)}$$

from node 2, using KCL

$$i_4 + 6 + 12 = i_3$$

$$i_4 - i_3 = -6 - 12$$

$$i_4 - i_3 = -18$$

$$i_4 = \frac{V_1 - V_2}{2}$$

$$i_3 = \frac{V_2 - 0}{4} = \frac{V_2}{4}$$

$$\Rightarrow \frac{V_1 - V_2}{2} - \frac{V_2}{4} = -18$$

$$= \frac{2(V_1 - V_2)}{4} - \frac{V_2}{4} = -18$$

$$\frac{2V_1 - 2V_2 - V_2}{4} = -18$$

$$2V_1 - 3V_2 = (-18 \times 4)$$

$$\Rightarrow 2V_1 - 3V_2 = -72 \quad ---- \text{(ii)}$$

$$8V_1 - 5V_2 = -120 \quad \text{---- (i)} \times 3$$
$$2V_1 - 3V_2 = -72 \quad \text{---- (ii)} \times 5$$

$$24V_1 - 15V_2 = -360$$
$$-\quad 10V_1 - 15V_2 = -360$$
$$14V_1 = 0$$
$$V_1 = 0$$

Substitute $V_1 = 0$ into any of the eqns

$$8(0) - 5V_2 = -120$$
$$-5V_2 = -120$$
$$V_2 = \frac{-120}{-5} = 24V$$

$V_1 = 0$ & $V_2 = 24V$

$$\therefore i_1 = \frac{V_1}{10} = \frac{0}{10}$$
$$= 0A$$

$$i_2 = \frac{V_1}{5} = \frac{0}{5}$$
$$= 0A$$

$$i_3 = \frac{V_2}{4} = \frac{24}{4}$$
$$= 6A$$

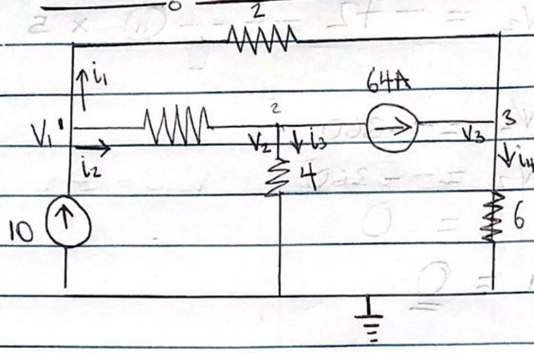
$$i_4 = \frac{V_1 - V_2}{2} = \frac{0 - 24}{2} = -12A$$

$\therefore V_1 = 0_V$, $V_2 = 24V$

$i_1 = 0A$ $\quad i_3 = 6A$

$i_2 = 0A$ $\quad i_4 = -12A$

Question 2 Solution



(a) For node 1; $\therefore i_1 + i_2 = 10$
using KCL

$$\frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3} = 10$$

(b) node 2; using KCL

$$\frac{V_1 - V_2}{3} = 64 + \frac{V_2}{1}$$

$$\therefore i_2 = 64 + i_3$$

(c) node 3; using KCL

$$\frac{V_3}{6} = 64 + \frac{V_1 - V_3}{2}$$

$$\therefore i_4 = 64 + i_1$$

Simplify;

$$10 = \tfrac{5}{6}V_1 - \tfrac{1}{3}V_2 - \tfrac{1}{2}V_3$$
$$64 = \tfrac{1}{3}V_1 - \tfrac{7}{12}V_2 + 0V_3$$
$$64 = -\tfrac{1}{2}V_1 - 0V_2 + \tfrac{2}{3}V_2$$

Using matrix form to solve;

$$\begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix} = \begin{bmatrix} 5/6 & -1/3 & -1/2 \\ 1/3 & -7/12 & 0 \\ -1/2 & 0 & 2/3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$b = A \quad x$$

$$x = A^{-1}b$$

$$A^{-1}b = \begin{bmatrix} 3.73 & -2.13 & 2.8 \\ 2.13 & -2.93 & 1.6 \\ 2.8 & -1.6 & 3.6 \end{bmatrix} \begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix}$$

$$x = \begin{bmatrix} 80 \\ -64 \\ 156 \end{bmatrix}$$

$\Rightarrow V_1 = 80v \quad V_2 = -64v \quad \& \quad V_3 = 156v$

Question 3 Solution

Using KCL @ node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$$\rightarrow \frac{V-21}{4} + \frac{V}{3} + \frac{V_1}{6} + \frac{V_1}{2} = 0$$

$$\Rightarrow \frac{3(V-21) + 4V + 2V_1 + 6V_1}{12} = 0$$

$$\Rightarrow \frac{3V - 63 + 4V + 2V_1 + 6V_1}{12} = 0$$

$7V + 8V_1 - 63 = 0$

$7V + 8V_1 = 63$ ------ (1)

Using KVL for loop 1

$\rightarrow -V - 9 + V_1 = 0$

$-V + V_1 = 9$ ------ (2)

$7V + 8V_1 = 63$
$-V + V_1 = 9$

Solving simultaneously,
$V = -0.6V$ $\quad V_1 = 8.4V$

(*) Current through the 3Ω resistor;
Using nodal analysis

$I_{3\Omega} = \frac{V-0}{3}$

$= \frac{-0.6-0}{3} = \frac{-0.6}{3}$

$\therefore I_{3\Omega} = -0.2A$ //

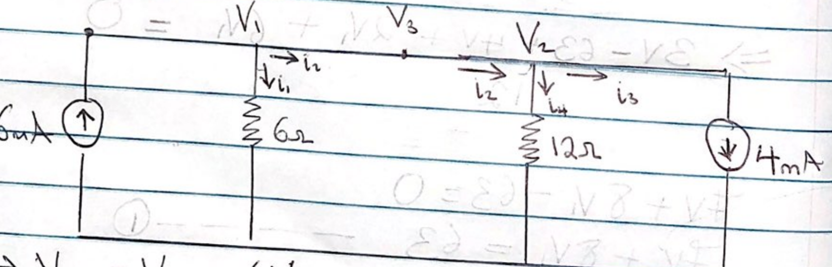
(*) Current through the 2Ω resistor,

$I_{2\Omega} = \frac{V_1 - 0}{2}$

$= \frac{8.4-0}{2} = 4.2$

$I_{2\Omega} = 4.2A$ //

Question 4 Solution



$\rightarrow V_1 - V_2 = 6V$

assume $V_1 - V_2 = i_2$

at node 1; using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1}{6} + (V_1 - V_2)$

$$6mA = \frac{V_1 + 6(V_1 - V_2)}{6}$$

$$6mA = \frac{V_1 + 6V_1 - 6V_2}{6}$$

$$36 = 7V_1 - 6V_2 \quad \text{----} \quad (i)$$

(*) at node 2 using KCL

$i_2 = i_4 + i_3$

$i_2 = i_4 + 4mA$

$\Rightarrow 4mA = i_2 - i_4$

$$4mA = (V_1 - V_2) - \frac{V_2}{12}$$

$$4mA = \frac{12(V_1 - V_2) - V_2}{12}$$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \quad \text{-----} \quad (ii)$

Solving eqn (i) & (ii) simultaneously, we have

$V_1 = 9.5V \quad V_2 = 5.1V$

Current through the 6Ω $(I_{6\Omega})$

$i_1 = \frac{V_1}{6}$

$= \frac{9.5}{6}$

$= 1.58A$ //

Current through the 12Ω $(I_{12\Omega})$

$i_4 = \frac{V_2}{12}$

$= \frac{5.1}{12} = 0.43A$

$i_2 = V_1 - V_2$

$= 9.5 - 5.1 = 4.4A$ //

$\therefore V_1 = 9.5V$, $V_2 = 5.1V$

$I_{6\Omega} = 1.58A$ & $I_{12\Omega} = 0.43A$