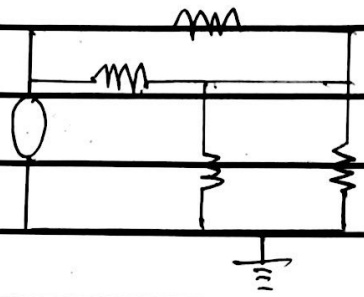
Ugah - Uche Chukwurenye

17/ENG04/071

ELECT/ELECT

Circuit Theory Assignment

① Find the voltages at node 1, 2 and 3 in the circuit below.



Node 1 KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3}$

multiply through by 6

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 3V_3 - 2V_2$ —— ①

At Node 2 KCL

$I_2 = i_3 + i_4$

$i_4 = i_2 - i_3$

$i_4 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$V_1 = 0V$ $V_2 = -24V$

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

multiply through by 7

$7i_8 = 4(V_1 - V_2) - 3(V_2 - 0)$

$7i_8 = 4V_1 - 7V_2$ —— ②

(3) Node 3 KCL

$6A + i_1 = 13$

$6A = 15 - 14$

$6A = \frac{V_3 - 6}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ — (3)

$5V_1 - 2V_2 - 3V_3 = 60$ — (1)

$4V_1 - 7V_2 + 0V_1 = 768$ — (2)

$-3V_1 + 0V_3 + 4V_3 = 384$

matrix Rep

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$ $V_2 = \frac{\Delta_2}{\Delta_2}$ $V_3 = \frac{\Delta_3}{\Delta_3}$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$V_2 = \Delta_2 =$

5	60	-3
4	768	0
-3	384	4

$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - (768 - 3))$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$V_3 =$

5	-2	60
4	-7	768
-3	0	384

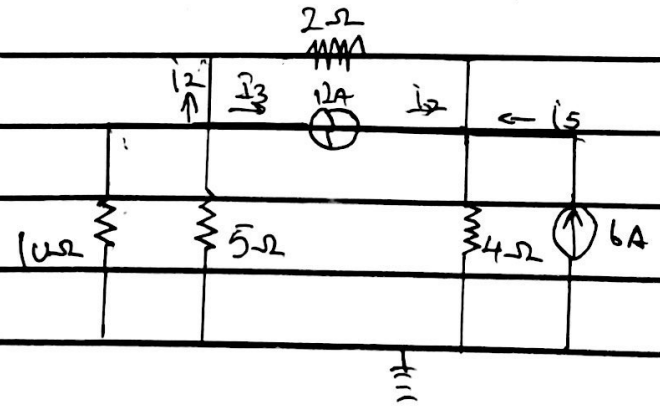
$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$

$= -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

$\therefore V_1 = 80V, \quad V_2 = -64V, \quad V_3 = 156V$

(21)



Node 1 KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_3}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ — (1)

Node 2

$i_3 + i_2 + i_5 = 16$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 + 48 = 2V_2$

$144 = -4V_1 + 6V_2$ — (2)

Using Elimination method

$120 = -8V_1 + 5V_2$ — (1) $\times 4$

$144 = -4V_1 + 6V_2$ — (2) $\times 8$

$-480 = 32V_1 + 20V_2$ — (iii)

$-1152 = 32V_1 - 48V_2$ — (iv)

Subtract eqn (3) from (4)

$-672 = 0 - 28V_2$

$-672 = -28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24\ V$

Sub V_2 into eq (2)

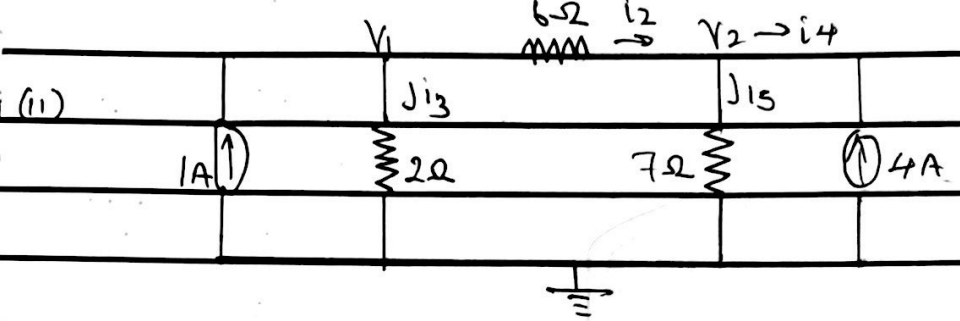
$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6(-24)}{-4}$

$V_1 = \frac{-144-144}{-4} = 0V$

$V_1 = 0V \quad V_2 = 24V$

$i_1 = 0A, \; i_2 = 0A, \; i_3 = 6A, \; i_4 = -12A$



Node 1

$i_1 = i_2 + i_3$

$i = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \quad -(1)$

Node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 6(14 + V_2)$

$7V_1 - 7V_2 = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \quad -(2)$

From equation (1)

$V_2 = 4V_1 - 6$

Sub $V_2 = 4V_1 - 6$ into eqn (2)

$168 = 7V_1 - 13(4V_1 - 0)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$\therefore V_1 = -2V$

Subs $V_1 = -2$ into Eqn (1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

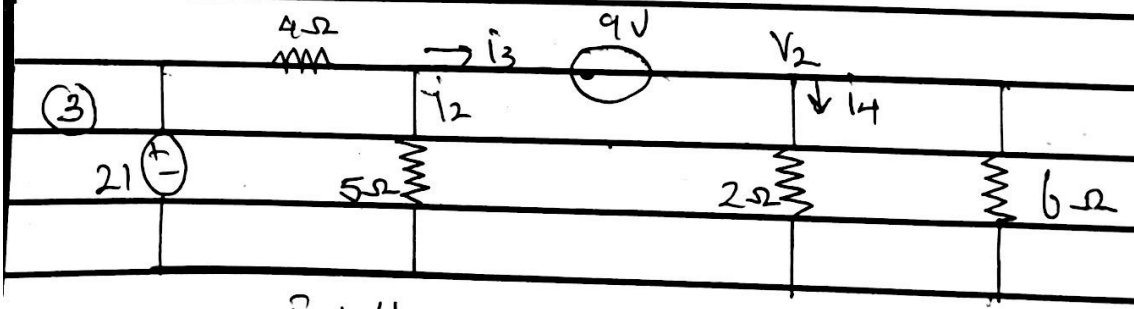
$V_1 = -2V$, $V_2 = -14V$

Current through the resistors.

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_3 = \frac{V_2}{7} = \frac{-14}{7} = -2A$



Find the current through the 3Ω and 2Ω resistors

Use KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0$ — (1)

taking KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ — (2)

$7V_1 + 8V_2 = 63$ — (1)

$-V_1 + V_2 = 9$

from eqn (2) $-V_1 + V_2 = 9$

$V_2 = 9 + V_1$ put into eq (1)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = 63 - 72$

$V_1 = -9/15$

$V_1 = -0.6V$

Sub $V_2 = -0.6$ in eq (1)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 9 - 0.6 = 8.4V$

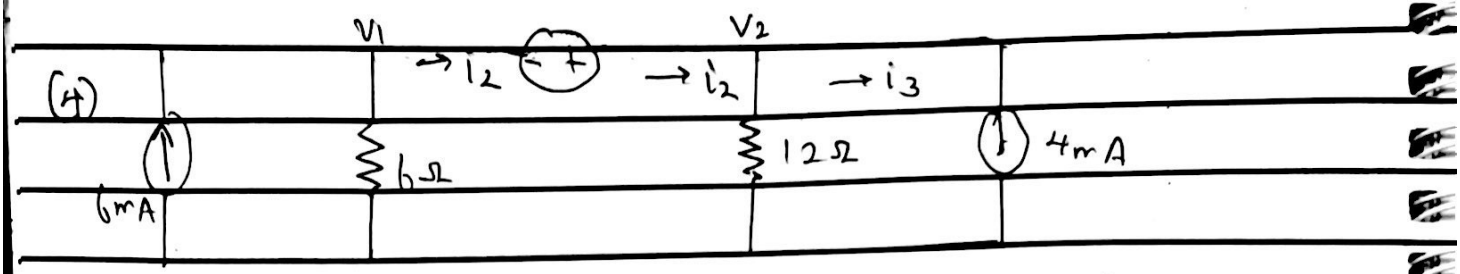
$\therefore V_1 = 0.6V \quad V_2 = 8.4V$

current through 3Ω and 2Ω

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$ at 3Ω

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$ at 2Ω

(4)



find the nodes at the voltage and the current through the 6Ω and 12Ω resister.

Tano $V_1 - V_2 = 6V \rightarrow i_2$

at node 1 Using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$31 = 7V_1 - 6V_2 \quad — (1)$

At Node 3

$i_2 = i_3 + i_4$

$\frac{V_1 - V_2}{12} = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \quad — (2)$

Solving V_1 and V_2 Simultaneosly we have

$V_1 = 9.5V$

$V_2 = 5.1V$

current through the 6Ω resister

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4mA$

Current through 12 Ω

$i_4 = \frac{V_2}{i_2} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V$ $V_2 = 5.1V$

$I_1 = 1.58A$ $I_4 = 0.43A$