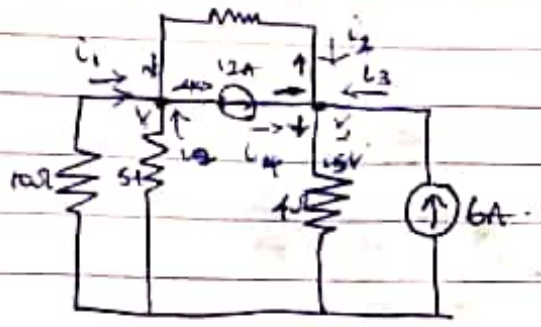
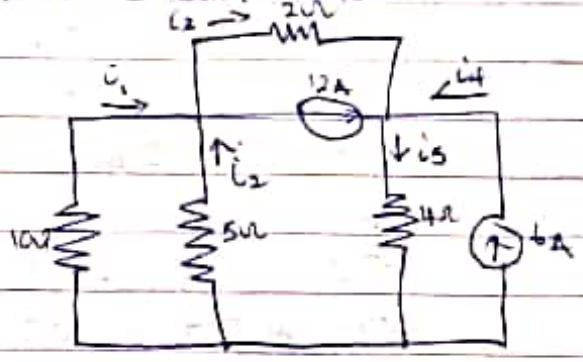
Name: Ubamgba Rita Chidum.
Matric No: 17/ENG04/069.
Department: Electrical Electronics Engineering.
EEE 323
Circuit theory. 2.

1) Find the voltages of nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below.





Solution.

At node 1.

$$\frac{-V_1}{10} - \frac{V_1}{5} = 12 + \frac{V_1 - V_2}{2}$$

$$\frac{-V_1 - 2V_1}{10} = 12 + \frac{V_1 - V_2}{2}$$

$$\frac{-3V_1}{10} = 12 + \frac{V_1 - V_2}{2}$$

$$-3V_1 = 120 + 5V_1 - 5V_2.$$

$$-120 = 8V_1 - 5V_2 \longrightarrow \text{equi}.$$

At node 2:

$$\frac{V_2 - 0}{4} = 12 + 6 + \frac{V_1 - V_2}{2}$$

$$\frac{V_2}{4} = 18 + \frac{V_1 - V_2}{2}$$

$$V_2 = 72 + 2V_1 - 2V_2.$$

$$-72 = 2V_1 - 3V_2. \longrightarrow \text{equii}$$

$-120 = 8V_1 - 5V_2 \longrightarrow$ equii

$-72 = 2V_1 - 3V_2 \longrightarrow$ equiii

Solving simultaneously:

$-120 = 8V_1 - 5V_2 \times 2$

$-72 = 2V_1 - 3V_2 \times 8$

$-240 = 16V_1 - 10V_2$

$-576 = 16V_1 - 24V_2$

$-336 = -14V_2$

$V_2 = \frac{-336}{-14}$

$V_2 = 24$

Sub V_2 in equ iii

$-72 = 2V_1 - 3(24)$

$-72 + 72 = 2V_1$

$V_1 = 0$

$i_1 = \frac{-V_1}{10}$

$i_1 = \frac{0}{10} = 0$

$i_3 = \frac{V_1 - V_2}{2}$

$= \frac{-24}{2}$

$= -12A$

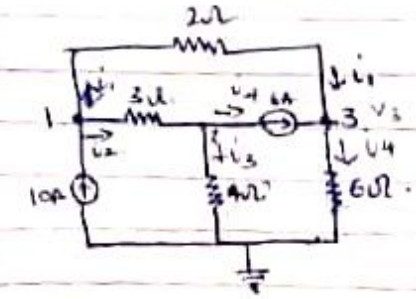
$i_2 = \frac{-V_1}{5}$

$= 0$

$i_4 = 6A$

$i_5 = \frac{V_2}{4}$

$= 0.$



ind the voltages at node 1, 2 and 3 in the circuit.

At node 1

Using KCL.

$10A = i_1 + i_2$.

$$10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$10 = \frac{3(V_1 - V_3) + 2(V_1 - V_2)}{6}$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \longrightarrow \text{equi}$$

At node 2,

Using KCL

$i_2 = i_3 + i_4$

$$\frac{V_1 - V_2}{3} = \frac{V_2 - 0}{4} + 6A$$

$$\frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4} = 6A$$

$$\frac{4(V_1 - V_2) - 3V_2}{12} = 6A$$

$$4V_1 - 7V_2 = 72 \longrightarrow \text{equii}$$

At node 3;

Using KCL

$i_4 = 6A + i_1$

$6A = i_4 + -i_1$

$$6A = \frac{V_2 - 0}{6} - \left(\frac{V_1 - V_3}{2}\right)$$

$$6A = \frac{V_3 - 3V_1 - 3V_3}{6}$$

2 | 6 | 2
6 | 3

$384 = V_3 - 3V_1 + 3V_3$

$384 = 4V_3 - 3V_1$ ⟶ equ iii

$60 = 5V_1 - 2V_2 - 3V_3$ ⟶ equ i

$768 = 4V_1 - 7V_2 - 0$ ⟶ equ ii

$384 = -3V_1 - 0 + 4V_3$ ⟶ equ iii

$$\begin{Bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{Bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 768 \\ 384 \end{Bmatrix}$$

$\Delta = 5(-28) - (-2(16)) - 3(-84)$

$= -140 + 32 + 63$

$= 95 - 140$

$= -45$

For Δ_1

$$\begin{Bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{Bmatrix}$$

$\Delta_1 = 60(-28) - (-2(3072)) - 3(2688)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80v$

For Δ_2

$$\Delta_2 = \begin{bmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{bmatrix}$$

$= 5(768 \times 4) - 60(16) - 3(1536 - (-2304))$

$= 15360 - 960 - 3(11520$

$= 15360 - 12480$

$= 2880$

$V_2 = \Delta_2/\Delta = \frac{2880}{-45} = -64v$

For Δ_3

$$\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5|$ $= 5(-2688) - (-2(1536 - (-2304))) + 60(-(21))$

$5|$ $= -13440 + 7680 - 1260$

$= -25140 + 7680 - 14700 + 7680$

$= -20460 - 7020$

$$V_3 = \frac{\Delta_3}{\Delta}$$

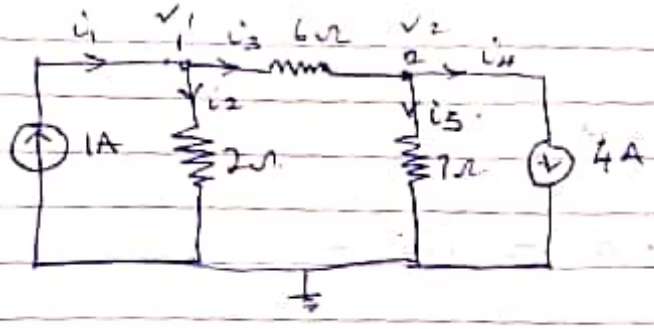
$$= \frac{-20460}{45} = \frac{-7020}{-45} = 156v$$

$V_1 = 80v$

$V_2 = -64v$

$V_3 = 156v$

3.



At node 1

$$1 = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{6}$$

$$1 = \frac{3V_1 + V_1 - V_2}{6}$$

$$6 = 4V_1 - V_2 \longrightarrow \text{equ i}$$

At node 2

$$\frac{V_1 - V_2}{6} = \frac{V_2 - 0}{7} + 4$$

$$4 = \frac{V_1 - V_2}{6} - \frac{V_2}{7}$$

$$4 = \frac{7V_1 - 7V_2 - 6V_2}{42}$$

$$168 = 7V_1 - 13V_2 \longrightarrow \text{equ ii}$$

$$6 = 4V_1 - V_2 \longrightarrow \text{equ i}$$

$$168 = 7V_1 - 13V_2 \longrightarrow \text{equ ii}$$

Solving simultaneously.

$4 \cdot 6 = 4V_1 - V_2 \longrightarrow$ equ i $\times 7$

$168 = 7V_1 - 13V_2 \longrightarrow$ equ ii $\times 4$

$$42 = 28V_1 - 7V_2$$

$$672 = 28V_1 - 52V_2$$

$$630 = -45V_2$$

$$V_2 = \frac{-630}{45}$$

$$V_2 = -14\text{ V}$$

Sub V_2 in equ i

$$6 = 4V_1 - (-14)$$

$$6 - 14 = 4V_1 \quad -8 = 4V_1 \quad V_1 = -2\text{ V}$$

$i_1 = 1A$

$i_2 = \frac{V_1}{2} = \frac{2}{2} = 1A$

$i_3 = \frac{V_1 - V_2}{6} = \frac{-2 - (-14)}{6} = \frac{12}{6} = 2A$

$i_4 = 4A$

$i_5 = \frac{V_2}{7}$

$= -\frac{14}{7}$

$= -2A$

$i_1 = 1A$

$i_2 = 7A$

$i_3 = 2A$

$i_4 = 4A$

$i_5 = 2A$