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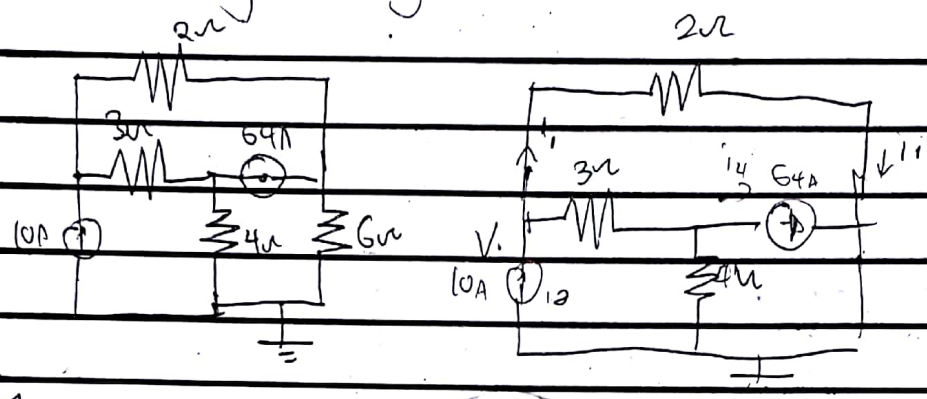
17/Eng04/041
Electrical/Electronics Engineering
Circuit Theory assignment II



node 1, KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 3V_3 - 2V_2 \quad -1)$

node 2 -

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2 \quad \ldots 1i)$

node 3

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \quad \text{---(iii)}$

Cramer rule

$5V_1 - 2V_2 - 3V_3 = 0$

$4V_1 - 7V_2 + 0V = 768$

$-3V_1 + 0V_2 + 4V_3 = 384$

convert to matrix

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta} \quad V_2 = \frac{\Delta_2}{\Delta} \quad V_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

find determine

$= 5(-28-0) - (-2(16-0)) + (-3(0-21))$

$\Rightarrow -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix}$$

determinant $= 60(-28-0) - (-768(-8-0) - 384(0-21))$

$= -1680 + 6144 - 8064$

$= -3600$

$\therefore V_1 = \frac{-3600}{-45} = 80V$

$V_2 = \Delta_2$ $\begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5((768 \times 4) - 0) - 4(240 - 1152) - 3(0 - 2304)$

$= +2880$

$V_2 = \frac{2880}{-45} = -64V$

$V_3 = \Delta_2$ $\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$

$= -7020$

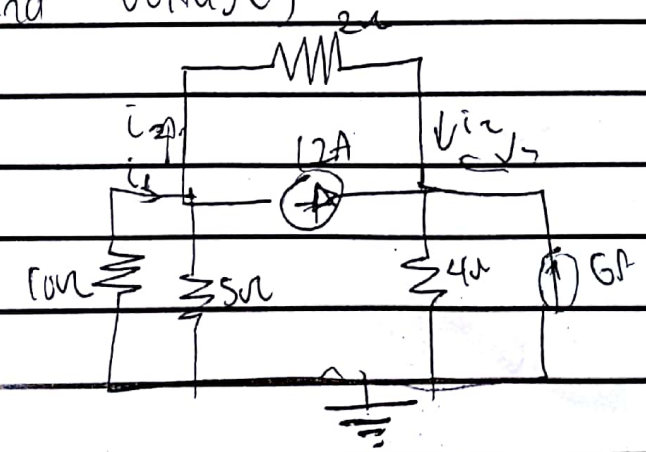
$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156$

$V_1 = 80V$

$V_2 = -64V$

$V_3 = 156V$

2) Find Voltage



At node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + i_2 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 - 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \ldots (I)$

node 2

$i_5 + i_2 + i_3 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \ldots (II)$

using elimination meth.

$120 = -8V_1 + 5V_2 \quad \ldots I)$

$144 = -4V_1 + 6V_2$

$-480 = 32V_1 - 20V_2 \quad III)$

$-1152 = 32V_1 - 48V_2 \quad IV)$

$672 = 0 - 28V_2$

$\therefore V_2 = 24V$

Subs V_2 in eq II)

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6(24)}{-4}$

$V_1 = 0$

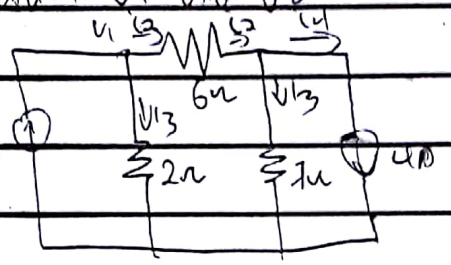
$V_1 = 0V, \quad V_2 = 24V$

$\therefore i_1 = 0A \quad i_4 = 12A$

$i_2 = 0A$

$i_3 = 6A$

1) Obtain V_1 and V_2



node 1

$i = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2} \Rightarrow 6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \quad \text{---(1)}$

At node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \quad \text{---(2)}$

from eqn(1) $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eq(2)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = -2V$

Subs $V_1 = -2$ in eq(1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$V_1 = -2V, \quad V_2 = -14V$

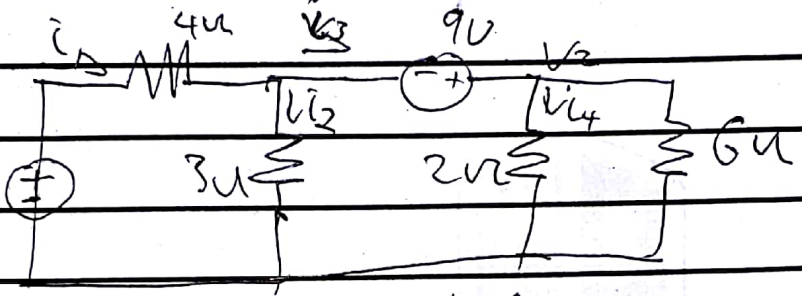
Current through the resistors

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A = 1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

3)



using KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 2}{4} + \frac{V_1}{3} + \frac{V_1 - V_2}{6} + \frac{V_1}{4}$

$\therefore 7V_1 + 8V_2 - 63 = 0$

$-V_1 - 9 + V_2 = 0$ — KVL for loop 3

$-V_1 + V_2 = 9 \quad -II)$

$7V_1 + 8V_2 = 63 \quad -I)$

$-V_1 + V_2 = 9 \quad -II)$

By simplifying

$V_2 = 9 + V_1 \quad -III)$

Substitute in I

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$\therefore 15V_1 = -9$

$V_1 = -0.6V$

Substitute $V_1 = -0.6$ in eqn II)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 8.4V$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

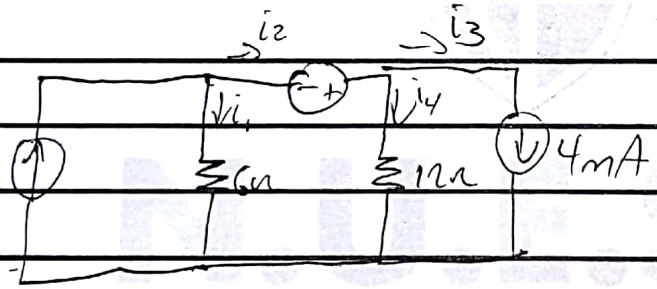
Current through the 3Ω resistor

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

$\frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4)



Find the node voltages and current

assume $V_1 - V_2 = 6V \Rightarrow i_2$

at node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad \cdots 1)$

At node 2

$i_2 = i_3 + i_4$

$\frac{V_1 - V_2}{6} = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \quad \text{--- 1)}$

Solving V_1 and V_2 simultaneously

$V_1 = 9.5V$ and $V_2 = 5.1V$

Current through 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore V_1 = 9.5V \qquad V_2 = 5.1V$

$i_1 = 1.58A \qquad i_2 = 0.43A$