

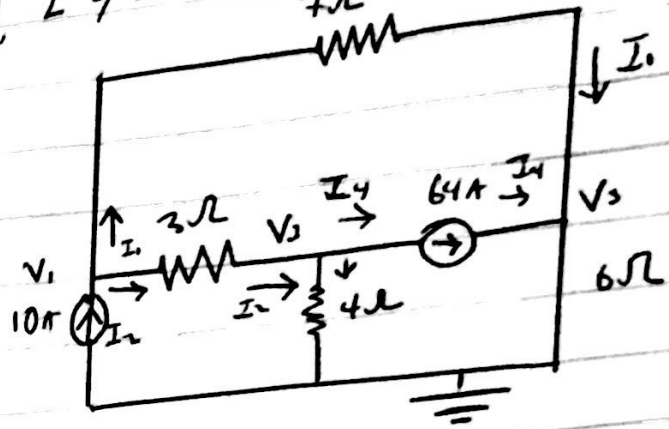
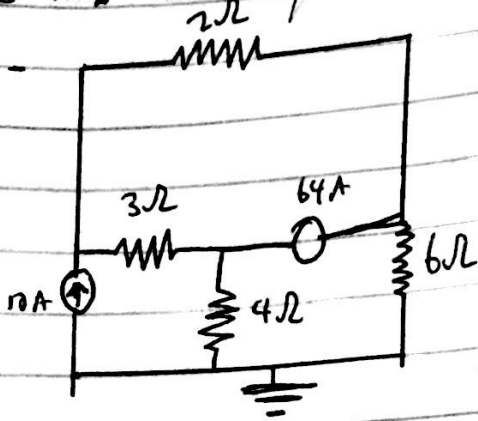
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17/ENG02/004

Computer ENGINEERING.

EEE 322

Find the voltages at nodes 1, 2 & 3 in the circuit below.



At node 1, KCL:

$$10 = I_1 + I_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_2) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_2 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \dots (1)$$

At node 2, KCL:

$$I_2 = I_3 + 64$$

$$64 = I_2 - I_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3[V_2 - 0]$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \dots (11)$$

At Node 3, KCL

$$64 + I_1 = I_5$$

$$64 = I_5 - I_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$$384 = -3V_1 + 4V_3 \dots \dots (iii)$$

Using Cramer's Rule

$$5V_1 - 2V_2 - 3V_3 = 60 \dots \dots (i)$$

$$4V_1 - 7V_2 = 768 \dots \dots (ii)$$

$$-3V_1 + 4V_3 = 384 \dots \dots (iii)$$

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\text{where } \Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix} \begin{matrix} + & - & + \\ + & - & + \end{matrix}$$

$$= 5(-28 - 0) + 2(16 + 0) - 3(0 - 21)$$

$$= -140 + 32 + 63$$

$$= -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28 - 0) - 768(-8 - 0) + 384(0 - 21)$$

$$= -168 + 6144 - 8064$$

$$= -360$$

$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$$

$$V_2 : \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ - & 4 & 768 \\ + & -3 & 384 \end{vmatrix}$$

$$= 5[(768 \times 4) - 0] - 4(160 \times 4) - (384 - 3) - 3(0 - 768 - 3)$$

$$= 2880$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

$$\text{for } V_3 : \begin{vmatrix} 5 & -2 & 60 \\ - & 4 & 768 \\ + & -3 & 384 \end{vmatrix}$$

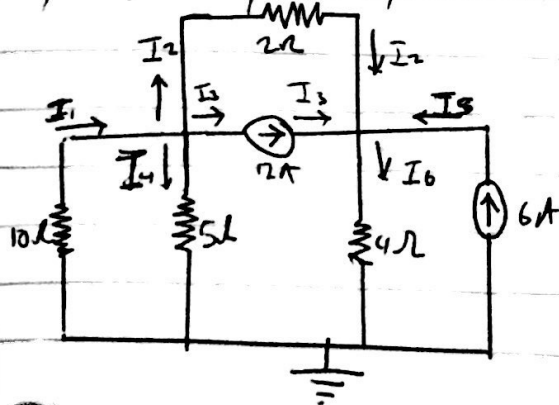
$$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 768) - (-7 \times 60)]$$

$$= -7020$$

$$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$$

Hence  $V_1 = 80V$ ,  $V_2 = -64V$ ,  $V_3 = 156V$

2). Find the Voltages at nodes 1 & 2 and determine the current flowing through the four resistors in the circuit below



At node 1, KCL

$$I_1 - I_2 + I_3 + I_4$$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 20 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 20 + 2V_1$$

$$120 = -8V_1 + 5V_2 \dots (i)$$

At node 2

$$I_3 + I_2 + I_3 = I_6$$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2(V_2)$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \dots (ii)$$

Using elimination method

$$120 = -8V_1 + 5V_2 \dots (i) \times -4$$

$$144 = -4V_1 + 6V_2 \dots (ii) \times -8$$

$$-480 = 32V_1 - 20V_2 \dots (iii)$$

$$-1152 = 32V_1 - 48V_2 \dots (iv)$$

Sub eqn (iii) from (iv)

$$-672 = 0 - 28V_2$$

$$V_2 = \frac{-672}{-28} \therefore V_2 = 24V$$

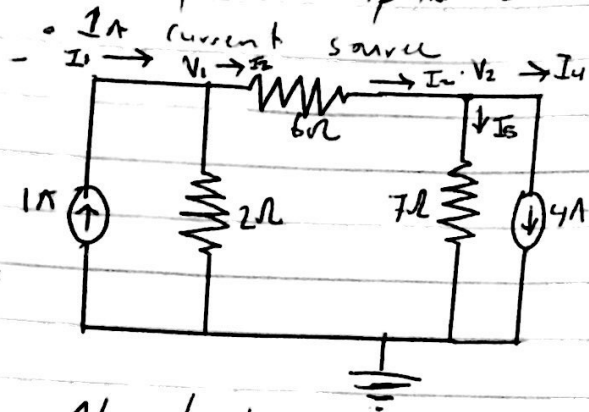
Sub eqn (ii) in eqn (ii)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6V_2}{-4}, \quad V_1 = \frac{144 - 6V_2}{-4} \therefore V_1 = 0$$

$$\therefore V_1 = 0V, \quad V_2 = 24V, \quad I_4 = 0A, \quad I_6 = 0A, \quad I_3 = 6A, \quad I_4 = -12A$$

ii] obtain  $V_1$  &  $V_2$  and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by



At node 1

$$I_1 = I_2 + I_3$$

$$I = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \dots (i)$$

At Node 2

$$I_2 = I_4 + I_3$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \dots (ii)$$

from eqn (i):  $V_2 = 4V_1 - 6$

Sub ( $4V_1 - 6$ ) for  $V_2$  in eqn (ii)

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45}$$

$$V_1 = -2V$$

Sub -2 for  $V_1$  in eqn (i)

$$6 = 4(-2) - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = -14V$$

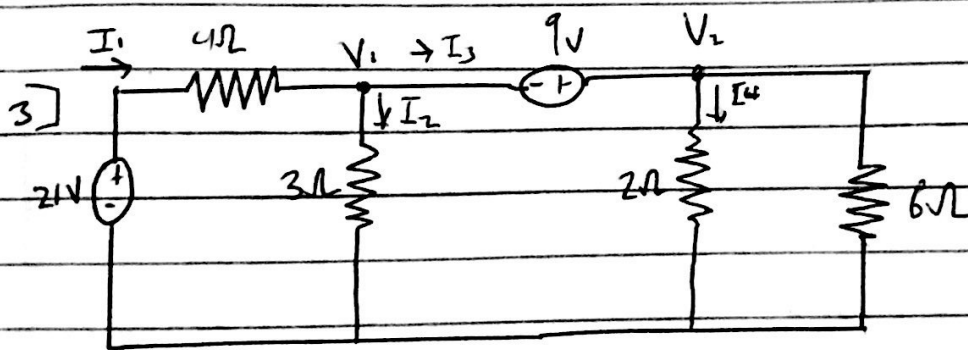
$$\therefore V_1 = -2V, V_2 = -14V$$

Current through the resistors

$$I_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$I_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$I_4 = \frac{V_2}{2} = \frac{-14}{2} = -7A$$



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_1 - V_2}{6} + \frac{V_2}{2} = 0$$

$$7V_1 + 8V_2 - 63 = 0 \dots (i)$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0, \quad -V_1 + V_2 = 9 \dots (ii)$$

$$7V_1 + 8V_2 = 63 \dots (i)$$

$$-V_1 + V_2 = 9 \dots (ii)$$

From (ii) Let  $V_2 = 1 + V_1$

Sub  $V_2 = 1 + V_1$  in eqn (i)

$$7V_1 + 8(1 + V_1) = 63$$

$$7V_1 + 8 + 8V_1 = 63$$

$$15V_1 = 55$$

$$V_1 = 3.67$$

Sub  $V_1 = 3.67$  in eqn (ii)

$$-(3.67) + V_2 = 9$$

$$0.67 + V_2 = 9$$

$$V_2 = 8.33$$

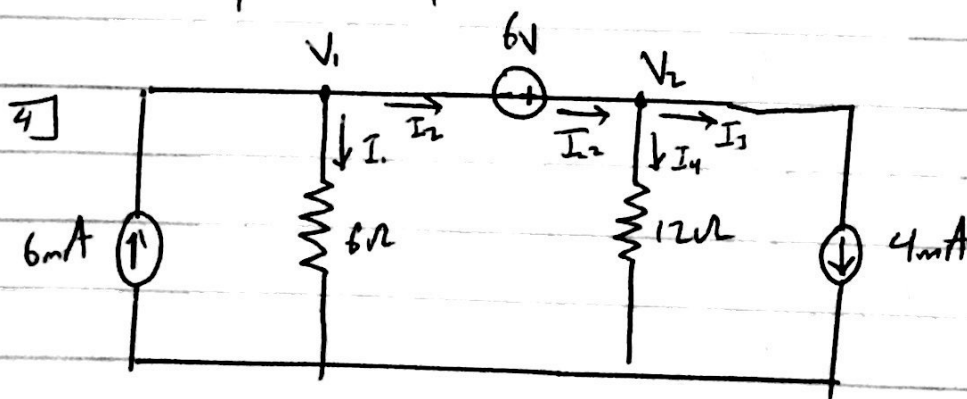
$$\therefore V_1 = 3.67 \text{ V and } V_2 = 8.33 \text{ V}$$

Current through the  $3\Omega$  resistor

$$I_2 = \frac{V_1}{3} = \frac{3.67}{3} = 1.22 \text{ A}$$

Current through the  $2\Omega$  resistor

$$I_4 = \frac{V_2}{4} = \frac{8.33}{4} = 2.08 \text{ A}$$



Find the node voltages and the current through the  $6\Omega$  &  $12\Omega$  resistors.  
 (Let's assume that  $V_1 - V_2 = 6V \rightarrow I_2$ )

At node 1, using KCL

$$6 \text{ mA} = I_1 + I_2$$

$$6 \text{ mA} = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$36 = V_1 + 6(V_1 - V_2)$$

$$36 = V_1 + 6V_1 - 6V_2$$

$$36 = 7V_1 - 6V_2 \dots (1)$$

At Node 2

$$I_2 = I_3 + I_4$$

$$V_1 - V_2 = 6 \text{ mA} + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 48 + V_2$$

$$48 = 12V_1 - 12V_2 + V_2$$

$$48 = 12V_1 - 11V_2 \dots (11)$$

Solving  $V_1$  &  $V_2$  simultaneously, we have

$$V_1 = 9.5 \text{ V} \quad \& \quad V_2 = 5.1 \text{ V}$$

$\therefore$  Current through the  $6 \Omega$  resistor

$$I_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58 \text{ A} \quad ; \quad I_2 = V_1 - V_2 = 9.5 - 5.1$$
$$I_2 = 4.4 \text{ A} //$$

Current through the  $12 \Omega$  resistor

$$I_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43 \text{ A}$$

$$\therefore V_1 = 9.5 \text{ V} \quad ; \quad V_2 = 5.1 \text{ V} //$$

$$I_1 = 1.58 \text{ A} \quad ; \quad I_2 = 0.43 \text{ A} //$$