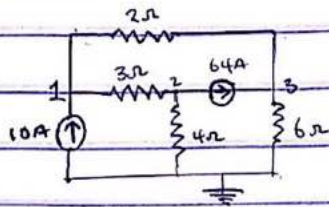


Question 1

Find the voltages at nodes 1, 2 and 3 in the circuit below



At node 1, KCL

$$10 = i_1 + i_2 \rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{--- (i)}$$

At node 2, KCL

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2 - 0$$

$$768 = 4V_1 - 7V_2 \quad \text{--- (ii)}$$

At Node 3

$$64 - i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$$384 = -3V_1 + 4V_3 \quad \text{--- (iii)}$$

Using Cramer's Rule

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \text{--- (1)}$$

$$4V_1 - 7V_2 = 748 \quad \text{--- (2)}$$

$$-3V_1 + 4V_3 = 384 \quad \text{--- (3)}$$

In matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 748 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\text{where } \Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$$\begin{aligned} &= 5(-28-0) + 2(16+0) - 3(0-28) \\ &= -140 + 32 + 84 \\ &= -45 \end{aligned}$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 748 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$$\begin{aligned} &= 60(-28-0) - 768(-8-0) - 384(0-28) \\ &= -1680 + 6144 - 8064 \\ &= -3600 \end{aligned}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80$$

$$V_2 = \Delta_2 = \begin{vmatrix} + & 5 & 60 & -3 \\ - & 4 & 768 & 0 \\ + & -3 & 384 & 4 \end{vmatrix}$$

$$= 5(768 - 0) - 4(240 + 1152) - 3(0 + 2244)$$

$$= 2880$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64$$

$$\text{For } V_3 \begin{vmatrix} + & 5 & -2 & 60 \\ - & 4 & -7 & 768 \\ + & -3 & 0 & 384 \end{vmatrix}$$

$$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$$

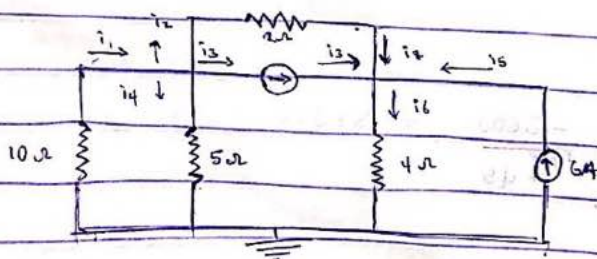
$$= -7020$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156$$

$$\therefore V_1 = 80V, V_2 = -64V, V_3 = 156V$$

Question 2

Find the voltages at nodes 1 and 2 and determine the current flowing through the four resistors in the circuit below



At Node 1, KCL

$$i_1 = i_2 + i_3 + i_4$$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + V_1 - V_2$$

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \quad \text{--- (i)}$$

At Node 2

$$i_3 + i_2 + i_5 = i_6$$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2(V_2)$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \quad \text{--- (ii)}$$

Using Elimination method

$$120 = -8V_1 + 5V_2 \quad \times 2$$

$$144 = -4V_1 + 6V_2 \quad \times 4$$

$$240 = -16V_1 + 10V_2 \quad \text{--- (iii)}$$

$$576 = -16V_1 + 24V_2 \quad \text{--- (iv)}$$

$$816 = 0 + 34V_2$$

$$V_2 = 24V$$

Sub V_2 in eqn (ii)

$$144 = -4V_1 + 24V$$

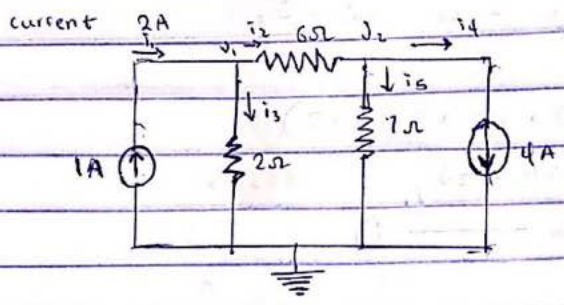
$$V_1 = 0$$

$$\therefore V_1 = 0V, \quad V_2 = 24V$$

$$i_1 = 0A, \quad i_2 = 0A, \quad i_3 = 6A, \quad i_4 = -12A$$

2 ii

Obtain i_1 and i_2 and the currents through the resistors for the circuit in example (i) if the $2A$ current source was replaced by a $1A$



Solution

At Node 1

$$i_1 = i_2 + i_5$$

$$1 = \frac{v_1 - v_2}{6} + \frac{v_1}{7}$$

$$6 = (v_1 - v_2) + 30$$

$$6 = 4v_1 - v_2 \quad \text{--- (i)}$$

At Node 2

$$i_2 = i_4 + i_5$$

$$\frac{v_1 - v_2}{6} = 4 + \frac{v_2}{7}$$

$$7(v_1 - v_2) = 168 + 6v_2$$

$$168 = 7v_1 - 13v_2 \quad \text{--- (ii)}$$

from eqn (i), $v_2 = 4v_1 - 6$

Sub v_2 in eqn (ii)

$$168 = 7v_1 - 13(4v_1 - 6)$$

$$168 = 7v_1 - 52v_1 + 78$$

$$168 - 78 = 7v_1 - 52v_1$$

$$90 = -45v_1$$

$$v_1 = -2V$$

sub $V_1 = -2$ in eqn (1)

$$6 = 4(-2) - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$= -14V$$

$$\therefore V_1 = -2V, V_2 = -14V$$

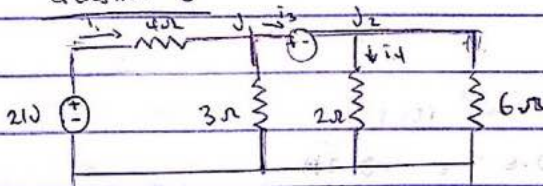
Current through the resistors

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$

Question 3



find the current through the 3Ω and 2Ω resistors

Solution

Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 2}{4} + \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

$$7V_1 + 8V_2 - 63 = 0 \quad \text{--- (1)}$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \text{--- (ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{---} \times 1$$

$$-7V_1 + V_2 = 9 \quad \text{---} \times 7$$

$$7V_1 + 8V_2 = 63 \quad \text{--- (iii)}$$

$$-7V_1 + 7V_2 = 63 \quad \text{--- (iv)}$$

$$0 + 15V_2 = 126$$

$$V_2 = \frac{126}{15} = 8.4$$

Sub V_2 in eqn (ii)

$$-7V_1 + 7(8.4) = 63$$

$$-7V_1 = 63 - 58.8$$

$$-7V_1 = 4.2$$

$$V_1 = -0.6$$

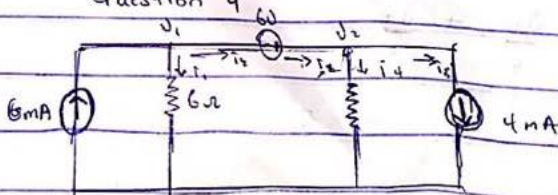
Current through the 2Ω resistor

$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2 \text{ A}$$

Current through the 2Ω resistor

$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 2.1 \text{ A}$$

Question 4



Find the node voltages and the currents through the 6Ω and 2Ω resistor.

Solution

Let say $V_1 - V_2 = 6V \rightarrow i_2$

At node 4 using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + \frac{(V_1 - V_2) \cdot 2}{12}$$

$$0.036 = V_1 + 6(V_1 - V_2)$$

$$0.036 = V_1 + 6V_1 - 6V_2$$

$$0.036 = 7V_1 - 6V_2$$

At Node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 0.048 + V_2$$

$$0.048 = 12V_1 - 12V_2 - V_2$$

$$0.048 = 12V_1 - 13V_2$$

$$7V_1 - 6V_2 = 0.036 \quad \times 12$$

$$12V_1 - 13V_2 = 0.048 \quad \times 7$$

$$84V_1 - 72V_2 = 0.432 \quad \text{--- (1)}$$

$$84V_1 - 91V_2 = 0.336 \quad \text{--- (2)}$$

$$0 + 19V_2 = 0.096$$

$$V_2 = 5.053 \times 10^{-3} V$$

Sub V_2 in eqn (1)

$$84V_1 - 0.4598 = 0.336$$

$$84V_1 = 0.336 + 0.4578$$

$$V_1 = \frac{0.7938}{84} = 9.474 \times 10^{-3} V$$

∴ Current through the 6Ω resistor

$$i_1 = \frac{V_1}{6} = \frac{9.474 \times 10^{-3}}{6} = 1.579 \times 10^{-3} \text{ A}$$

Current through the 12Ω resistor

$$i_4 = \frac{V_2}{12} = \frac{5.053 \times 10^{-3}}{12} = 4.212 \times 10^{-4} \text{ A}$$

$$V_1 = 9.474 \times 10^{-3} \text{ V}$$

$$V_2 = 5.053 \times 10^{-3} \text{ V}$$

$$i_1 = 1.579 \times 10^{-3} \text{ A}$$

$$i_4 = 4.212 \times 10^{-4} \text{ A}$$