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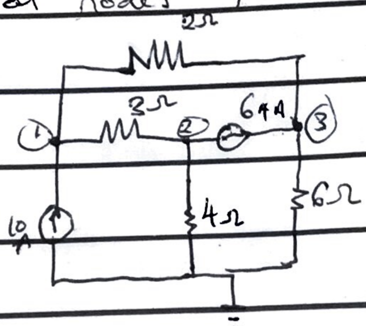
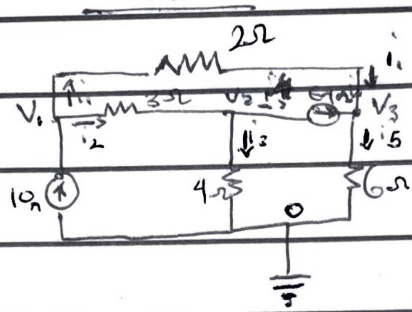
MAT NO: 17/ENG 04/051 COURSE CODE: ENG 322

1) Find the voltages at nodes 1, 2 and 3 in the circuit below



Soln



From the diagramatic expression above, at node 1, using KCL

$10A = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$\Rightarrow 10 = \frac{3(V_1 - V_3) + 2(V_1 - V_2)}{6}$

$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2) \Rightarrow 60 = 2V_1 + 3V_1 - 2V_2 - 3V_3$

$\Rightarrow 60 = 5V_1 - 2V_2 - 3V_3$ — (i)

at node 2, using KCL

$i_2 = i_3 + 64 \quad \therefore 64 = i_2 - i_3 \Rightarrow 64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$\Rightarrow 64 = \frac{4(V_1 - V_2) - 3(V_2 - 0)}{12} \Rightarrow 768 = 4(V_1 - V_2) - 3V_2$

$\Rightarrow 768 = 4V_1 - 4V_2 - 3V_2 \Rightarrow 768 = 4V_1 - 7V_2$ — (ii)

at node 3, using KCL

$64A + i_1 = i_5 \quad \therefore 64A = i_5 - i_1$

$\therefore 64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2} \Rightarrow 64 = \frac{2(V_3 - 0) - 6(V_1 - V_3)}{12}$

$\Rightarrow 768 = 2V_3 - 6V_1 + 6V_3 \Rightarrow 768 = 8V_3 - 6V_1$

dividing through by 2 we have $384 = 4V_3 - 3V_1$ — (iii)

Using Cramer's Rule we have;

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \ldots (i)$$

$$4V_1 - 7V_2 = 768 \quad \ldots (ii)$$

$$-3V_1 \quad 4V_3 = 384 \quad \ldots (iii)$$

In Matrix Representation we have

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

Where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$ thus we have

$$\Rightarrow 5(-28-0) + 2(16+0) - 3(\cancel{20}\ \cancel{+}21)$$

$$= -140 + 32 + 63$$

$$= -45$$

For $\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$$\Rightarrow 60(-28-0) - 768(-8-0) + 384(0-21)$$

$$= -1680 + 6144 - 8064$$

$$= -3600$$

$$\therefore V_1 = \Delta_1/\Delta = -3600/-45 = 80V$$

For $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$$\Rightarrow 5(3072-0) - 4(240+1152) - 3(0+2304)$$

$$= 15{,}360 - 5568 - 6912$$

$$= 2880$$

$$\therefore V_2 = \Delta_2/\Delta = 2880/-45 = -64V$$

For $\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

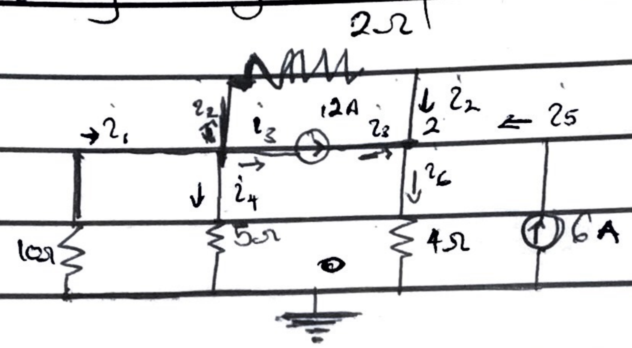
$$\Rightarrow 5((-7\times384)-0) - 4((-2\times384)-0) - 3((-2\times768) - (-7\times60))$$

$$= -7020$$

$$\therefore V_3 = \Delta_3/\Delta = -7020/-45 = 156V.$$

$$\therefore V_1 = 80V, \quad V_2 = -64V, \ \& \ V_3 = 156V.$$

Find the voltages at nodes 1 and 2 and also determine the currents flowing through the four resistors in the circuit below.



Soln

Using the KCL, at node 1 we have.

$$i_1 = i_2 + i_3 + i_4 \equiv \frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$$-V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1 - 0 \equiv 120 = -8V_1 + 5V_2 \quad \ldots (i)$$

at node 2

$$i_3 + i_2 + i_5 = i_6 \equiv 12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$\cancel{48 + 2(V_1 - V_2) + 24 = 2(V_2)}$$

$$\cancel{48 + 2V_1 - 2V_2 + 24 = 2V_2}$$

$$48 + 2V_1 - 2V_2 + 24 = V_2$$

$$72 + 2V_1 - 3V_2 = 0$$

$$\therefore -2V_1 + 3V_2 = 72 \quad -(ii)$$

Eliminating we have $=$

$$120 = -8V_1 + 5V_2 \quad (i) \quad \times 2$$

$$72 = -2V_1 + 3V_2 \quad (ii) \quad \times 8$$

$$240 = -16V_1 + 10V_2 \quad (iii)$$

$$576 = -16V_1 + 24V_2 \quad (iv) \text{ Subtracting iii from iv}$$

$$-336 = 0 - 14V_2$$

$$\therefore V_2 = \frac{-336}{-14} = 24V$$

Substituting V_2 into equation (i)

$120 = -8V_1 + 5(24)$

$\therefore \quad 120 = -8V_1 + 120$

$\therefore \quad 120 - 120 = -8V_1$

$\therefore \quad V_1 = 0/-8 = 0$

$\therefore V_1 = 0$ & $V_2 = 24\,V.$ using these values

$\cancel{i_1 = \frac{0 - 24}{10}} \quad \frac{0-0}{10} = \cancel{\frac{0-0}{10} = 0}$

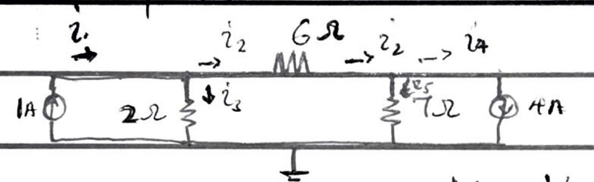
$\cancel{i_2 = \frac{V_1 - V_2}{2}} = \cancel{\frac{0-24}{2}} = -12$ ~~thus~~ $\cancel{0}$.

$i_3 = 6A$

$i_4 = -12A$

$i_1 = 0A,\ i_2 = 0A,\ i_3 = 6A,\ i_4 = -12A$

Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current source.



At node 1 $\quad i_1 = i_2 + i_3$

$1A = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$ multiplying through by 12

$12 = 2V_1 - 2V_2 + 6V_1$

$\Rightarrow 12 = 8V_1 - 2V_2$ ÷ by 2 $\Rightarrow 6 = 4V_1 - V_2$ — (i)

at node 2 $\Rightarrow i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2 \Rightarrow 7V_1 - 7V_2 = 168 + 6V_2$

$\therefore 168 = 7V_1 - 7V_2 - 6V_2 \rightarrow 168 = 7V_1 - 13V_2$ — (ii)

from equ 1 $\quad V_2 = 4V_1 - 6$ — (iii)

Sub equ 3 into equ (ii)

$\therefore \quad 168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$168 - 78 = -45V_1$

$90 = -45V_1 \quad \therefore V_1 = 90/-45 = -2V$

Substituting $V_1 = -2V$ in eq. (i)

$6 = 4(-2) - V_2 \Rightarrow 6 = -8 - V_2$

$\therefore V_2 = -14V$

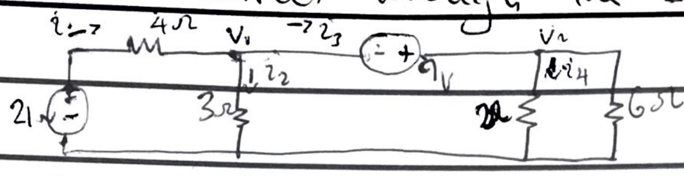
Current through the resistors

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$$

$$i_4 = V_2/7 = -14/7 = -2A$$

$$i_3 = V_1/2 = -2/2 = -1A$$

3) Find the current through the 3Ω and 2Ω resistors



Using KCL at Node 1 $\quad i_1 + i_2 + i_3 + i_4 = 0$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2} \quad \Rightarrow 7V_1 + 8V_2 - 63 = 0 \quad \text{(i)}$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$\therefore V_2 - V_1 = 9 \quad \text{--- (ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{— (i)}$$
$$-V_1 + V_2 = 9 \quad \text{— (ii)}$$

from (ii) Using $V_2 = 9 + V_1$

Substituting $V_2 = 9 + V_1$ in equ (i)

$$7V_1 + 8(9 + V_1) = 63$$
$$7V_1 + 72 + 8V_1 = 63$$
$$15V_1 = -9$$
$$V_1 = -9/15 = -0.6V$$

Subs V_1 into equ 2

$$-(-0.6) + V_2 = 9$$
$$9 - 0.6 = V_2 \Rightarrow V_2 = 8.4V$$

Thus current through the 3Ω resistor;

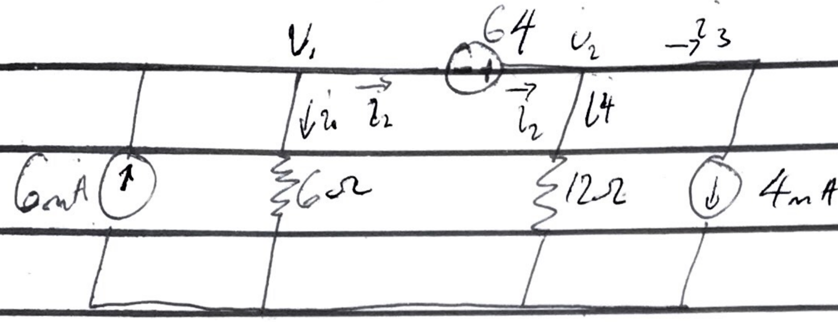
$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A.$$

Current through the 2Ω resistor

$$i_4 = V_2/4 = 8.4/4 = 4.2A.$$

4) Find the node voltages and the currents through the 6Ω and 12Ω resistor

assuming $V_1 - V_2 = 6\,V \Rightarrow i_2$



at node 1 using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad - (i)$

At node 2

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12} \Rightarrow 12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \quad - (ii)$

Solving V_1 and V_2 simultaneously, we have

$V_1 = 9.5V$ and $V_2 = 5.1V$.

Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$, $i_2 = V_1 - V_2 = 9.5 - 5.1$
$= 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore V_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_2 = 0.43A$