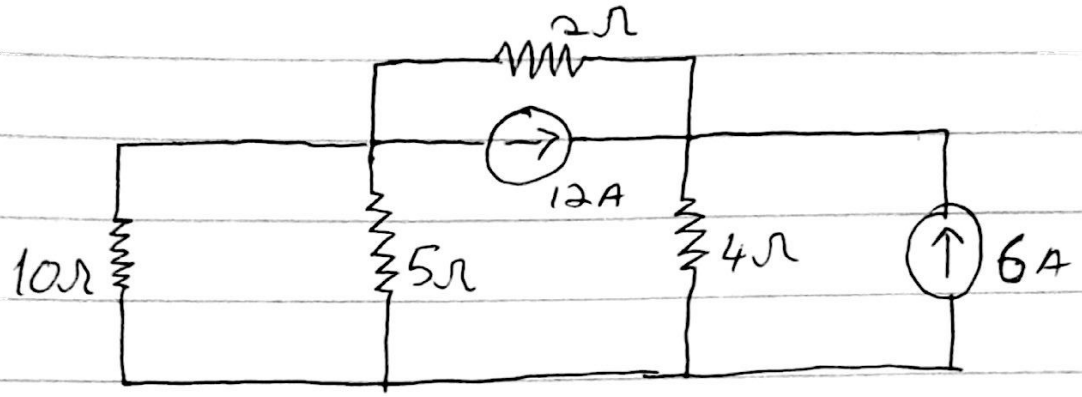
UGWUJA OGBONNA 17/ENG04/072

Circuit theory

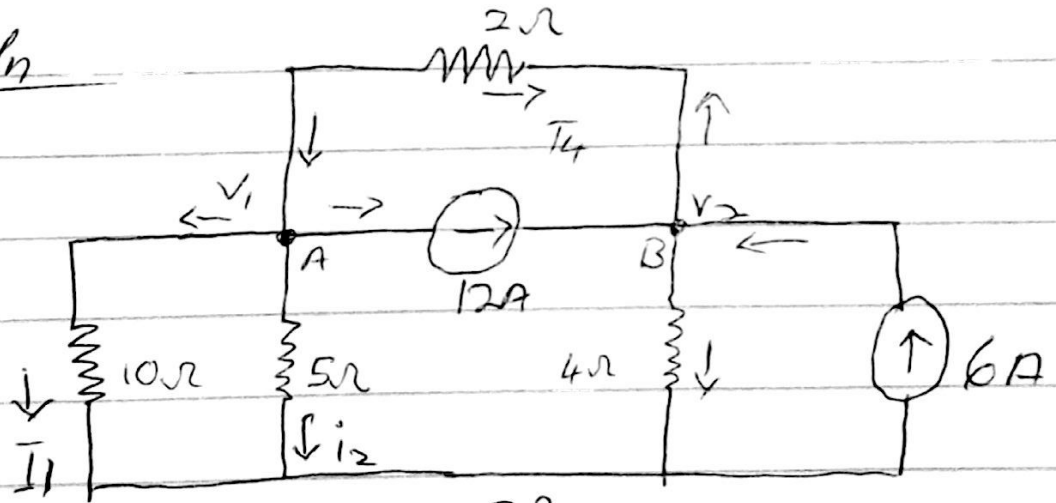
(1)

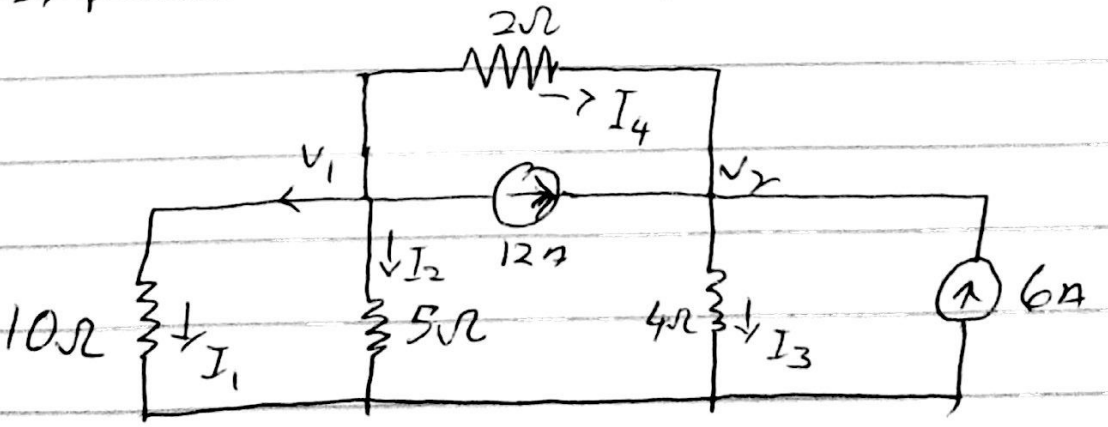


I Find the voltages at node 1 and 2

II Find the current flowing through the four resistors.

Soln





2

from node 1; using KCL

$i_1 + i_2 + i_4 = -12A$

$i_1 = \frac{V_1 - 0}{10} = \frac{V_1}{10}$

$i_2 = \frac{V_1 - 0}{5} = \frac{V_1}{5}$

$i_4 = \frac{V_1 - V_2}{2}$

$\frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -12$

$= \frac{V_1}{10} + \frac{2V_1}{10} + \frac{5(V_1 - V_2)}{10} = -12$

$= \frac{V_1 + 2V_1 + 5V_1 - 5V_2}{10} = -12$

$\frac{8V_1 - 5V_2}{10} = -12$

$\Rightarrow 8V_1 - 5V_2 = -120$

Applying KCL in node 2

$i_4 + 6 + 12 = i_3$

$i_4 - i_3 = -18$

$i_4 = \frac{V_1 - V_2}{2}$ and $i_3 = \frac{V_2 - 0}{4} = \frac{V_2}{4}$

$$\Rightarrow \frac{V_1 - V_2}{2} - \frac{V_2}{4} = -18$$

$$\frac{2(V_1 - V_2)}{4} - \frac{V_2}{4} = -18$$

$$\frac{2V_1 - 2V_2 - V_2}{4} = -18$$

$$\frac{2V_1 - 3V_2}{4} = -18$$

$$2V_1 - 3V_2 = -72 \quad ----(1)$$

$$8V_1 - 5V_2 = -120$$

$$2V_1 - 3V_2 = -72$$

Using Calculator to solve simultaneously

$V_1 = 0V$ & $V_2 = 24V$

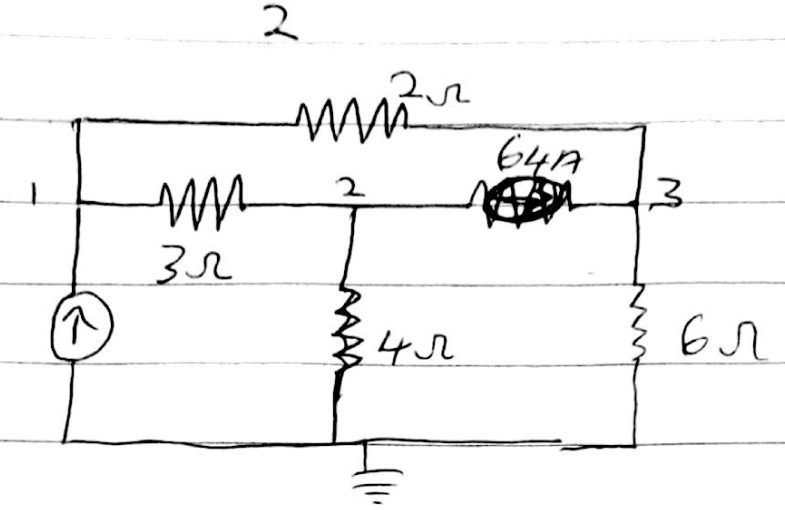
$$i_1 = \frac{V_1}{10} = \frac{0}{10} = 0A$$

$$i_2 = \frac{V_1}{5} = \frac{0}{5} = 0A$$

$$i_3 = \frac{V_2}{4} = \frac{24}{4} = 6A$$

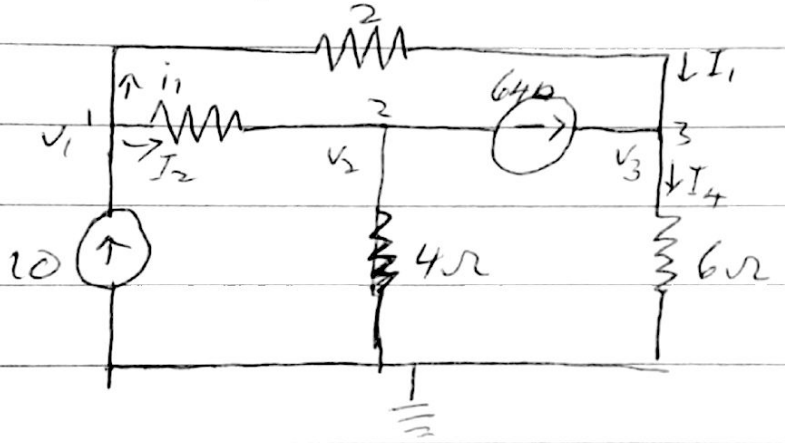
$$i_4 = \frac{V_1 - V_2}{2} = \frac{0 - 24}{2} = \frac{-24}{2} = -12A$$

4



Find the voltages at node 1, 2 & 3

Soln



(i) at node 1; using KCL $10 = i_1 + i_2$

$$10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

at node 2; using KCL : $i_2 = 64 + i_3$

$$\frac{V_1 - V_2}{3} = 64 + \frac{V_2}{4}$$

at node 3 using KCL; $i_4 = 64 + i_1$

$$\frac{V_3}{6} = 64 + \frac{V_2 - V_3}{2}$$

$10 = \frac{5}{6}V_1 - \frac{1}{3}V_2 - \frac{1}{2}V_3$

$64 = \frac{1}{3}V_1 - \frac{7}{12}V_2 + 0V_3$

$64 = -\frac{1}{2}V_1 - 0 + \frac{2}{3}V_3$

Using matrix method

$$\begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix} = \begin{bmatrix} 5/6 & -1/3 & -1/2 \\ 1/3 & -7/12 & 0 \\ 1/2 & 0 & 2/3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

A B C

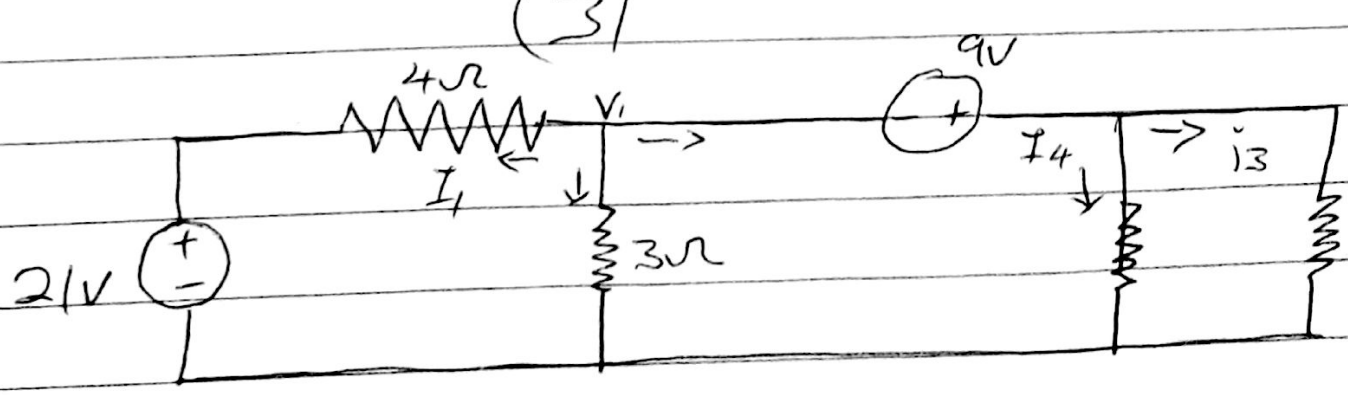
$C = B^{-1}A$

$$B^{-1}A = \begin{bmatrix} 3.73 & -2.13 & 2.8 \\ 2.13 & -2.93 & 1.6 \\ 2.8 & -1.6 & 3.6 \end{bmatrix} \begin{bmatrix} 10 \\ 64 \\ 64 \end{bmatrix}$$

$$C = \begin{bmatrix} 80 \\ -64 \\ 156 \end{bmatrix}$$

$\therefore V_1 = 80V, \quad V_2 = -64V \quad V_3 = 156V$

(3)



6

$2||6 = \frac{2 \times 6}{2+6} = \frac{12}{8} = 1.5$

find the current through 3Ω resistor

using KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V}{3} + \frac{V_1}{6} + \frac{V_1}{2} = 0$

$\Rightarrow \frac{3(V-21) + 4V + 2V_1 + 6V_1}{12} = 0$

$= \frac{3V - 63 + 4V + 2V_1 + 6V_1}{12} = 0$

$= 7V + 8V_1 - 63 = 0$

$7V + 8V_1 = 63 \quad ---(1)$

Using KVL at loop 1

$\rightarrow -V - 9 + V_1 = 0$

$-V + V_1 = 9 \quad ----(2)$

$7V + 8V_1 = 63$

$-V + V_1 = 9$

using my calculator to solve simultaneously

$V = -0.6V \; ; \; V_1 = 8.4V$

* The current through the 3Ω resistor;

Using nodal analysis

$I_{3\Omega} = \frac{V - 0}{3}$

$$= \frac{-0.6-0}{3} = \frac{-0.6}{3}$$

$$\therefore I_{3\Omega} = -0.2A$$

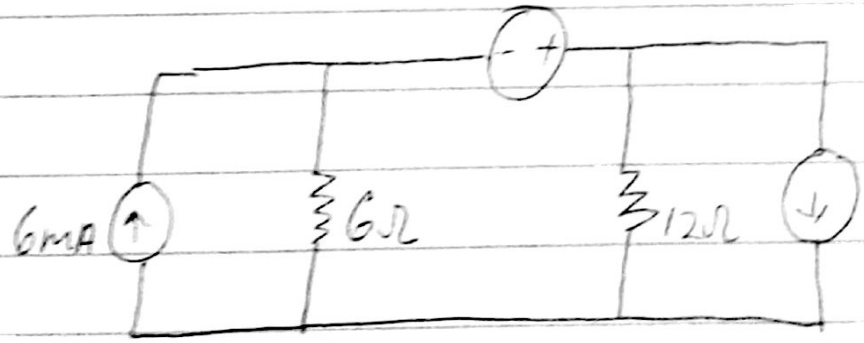
And the current through the 2Ω resistor

$$I_{2\Omega} = \frac{V_1 - 0}{2}$$

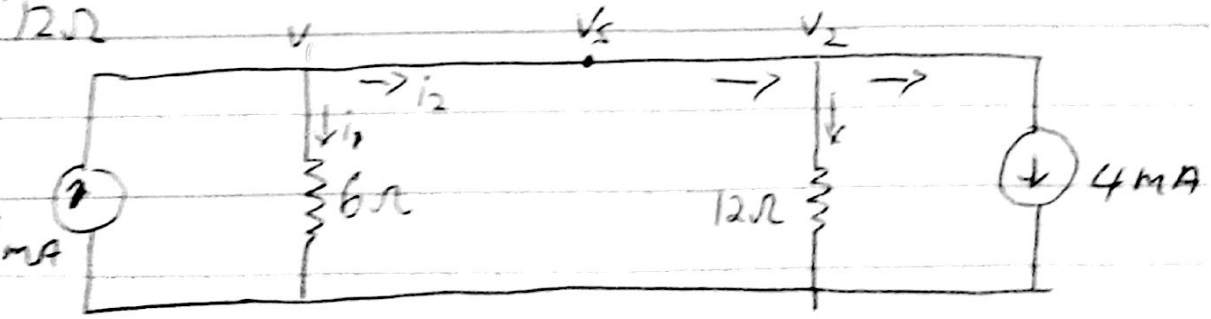
$$= \frac{8.4-0}{2} = \frac{8.4}{2}$$

$$I_{2\Omega} = 4.2A$$

(4)



find the voltages and currents through 6Ω &



8

→ $V_1 - V_2 = 6V$

Assume $V_1 - V_2 = i_2$

Using KCL at node 1

$6mA = i_1 + i_2$

$6mA = \frac{V_1}{6} + (V_1 - V_2$

$6mA = \frac{V_1 + 6(V_1 - V_2)}{6}$

$6mA = \frac{V_1 + 6V_1 - 6V_2}{6}$

$36mA = 7V_1 - 6V_2$

At node 2; $i_2 = i_4 + i_3$ $\quad i_3 = 4mA$

→ $i_2 = i_4 + 4mA$ $\quad \therefore 4mA = i_2 - i_4$

$4mA = (V_1 - V_2) - \frac{V_2}{12}$

$4mA = \frac{12(V_1 - V_2) - V_2}{12}$

$48mA = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2$ ---- (2)

Solving Simultaneously

$V_1 = 9.5V$, $V_2 = 5.1V$

Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$I_2 = V_1 - V_2$

$= 9.5 - 5.1$

$= 4.4A$

$i_4 = \frac{V_2}{I_2} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V, \quad V_2 = 5.1V$

$i_4 = 1.58 \quad i_{12} = 0.43A$