ELENDU JUSTICE CHIRINGO

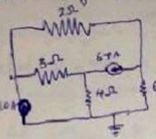
17/ENG02/021

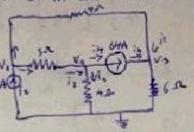
COMPUTER ENGINEERING

300L

ENG 322 ASSIGNMENT

1) Find the voltage at nodes 1, 2 and 3 in the circuit below





At node 1, KCL:

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_3 \quad \cdots (i)$

At node 2, KCL:

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2 \cdots (ii)$

At node 3, KCL

$64 + i_1 = i_3$

$64 = i_3 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \cdots (iii)$

Using Cramer's Rule

$5V_1 - 2V_2 - 3V_3 = 60 \quad \dots (i)$

$4V_1 - 7V_2 = 768 \quad \dots (ii)$

$-3V_1 + 4V_3 = 384 \quad \dots (iii)$

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} \begin{matrix} +\\ - \\ + \end{matrix}$

$60(-28-0) - 768(-8-0) + 384(0-21)$

$-1680 + 6144 - 8064$

-3600

$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For V_2: $\Delta_2 \begin{matrix} + \\ - \\ + \end{matrix} \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5((768\times4)-0) - 4((60\times4)-(384\cdot-3)) - 3(0-(768\cdot-3$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

For V_3 $= \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

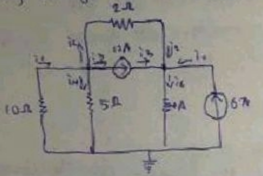
$= 5((-7\times384)-0) - 4((-2\times384-0) - 3((-2\times768)-(-7\times60))$

$= -7020$

$\therefore V_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{-7020}{-45} = 156V$

Therefore, $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

2 Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below



At Node 1; KCL

$i_1 = I_2 + i_3 + i_4$

$\dfrac{V_0 - V_1}{10} = \dfrac{V_1 - V_2}{2} + 12 + \dfrac{V_1 - V_0}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \cdots (i)$

At Node 2

$i_C = i_3 + i_2 + i_6$

$\dfrac{V_2 - 0}{4} = 12 + \dfrac{V_1 - V_2}{2} + 6$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \cdots (ii)$

Using elimination method

From eqn (1).

$120 = -8V_1 + 5V_2 \cdots (i) \quad \times -4$

$144 = -4V_1 + 6V_2 \cdots (ii) \quad \times -8$

$-480 = 32V_1 - 20V_2 \cdots (iii)$

$-1152 = 32V_1 - 48V_2 \cdots (iv)$

Substract eqn (iii) from (iv)

$-672 = 0 - 28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24V$

Subs $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6V_2}{-4}$

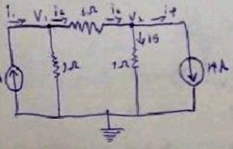
$V_1 = \frac{144 - 6V_2}{-4}$

$V_1 = 0$

$\therefore \quad V_1 = 0V, \; V_2 = 24V$

$i_1 = 0A, \; i_2 = 0A, \; i_3 = 6A, \; i_4 = -12A$

ii obtain v_1 and v_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current source.



At Node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_2}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \cdots (i)$

At Node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \cdots (ii)$

From eqn (i).

$V_2 = 4V_1 - 6$

Substitute $V_2 = 4V_1 - 6$ in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Substitute $V_1 = -2$ in eqn (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$\therefore V_1 = -2V,\ V_2 = -14V$

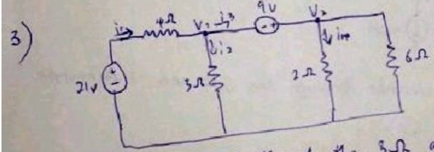
Current through the resistors

$i_2 = \dfrac{V_1 - V_1}{6} = \dfrac{-2+14}{6} = 2A$

$i_3 = \dfrac{V_1}{2} = \dfrac{-2}{2} = -1A$

$i_5 = \dfrac{V_2}{7} = \dfrac{-14}{7} = -2A$

3)



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\dfrac{V_1 - 21}{4} - \dfrac{V_1}{3} + \dfrac{V_2}{6} + \dfrac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0 \quad \cdots (i)$

Using KVL for Loop 1

$-V - 9 + V_2 = 0$

$-V_1 + V_2 = 9 \cdots (ii)$

$7V_1 + 8V_2 = 63 \quad \cdots (i)$

$-V_1 + V_2 = 9 \quad \cdots (ii)$

ii) let $V_2 = 9 + V_1$

Substitute $V_2 = 9 + V_1$ in eqn (i)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V_1 = -0.6V$

Substitute $V_1 = -0.6$ in eqn (ii)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 8.4V$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

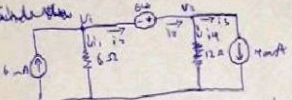
Current through the 3Ω resistor

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4) ~~Find the~~



Find the node voltage and the currents through the 6Ω and 12Ω resistor

lets assume $V_1 - V_2 = 6V = i_2$

At node 1, using KCL

$6mA = i + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \cdots (i)$

At Node 2

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2 \cdots (ii)$

Solving V_1 and V_2 simultaneously, we have

$V_1 = 9.5v$ and $V_2 = 5.1v$

$\therefore$ current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$.

$i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore$ $V_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_4 = 0.43A$.