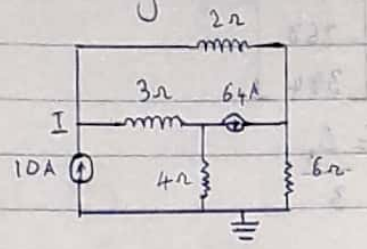
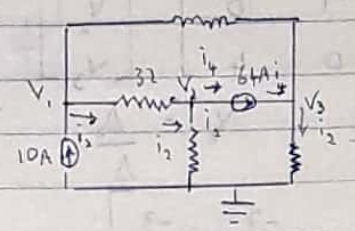
ASITA OBONISO

17/ENG06/014 MECHATRONICS ENGINEERING

CIRCUIT THEORY

1 Find the voltages at node 1, 2 and 3 in the circuit below





Nodal KCL

$10 = i_1 + i_2$

$\Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_2) + 2(V_1 - V_2)$ [multiplying through by 6]

$60 = 3V_1 - 3V_2 + 2V_1 - 2V_2$

$60 = 5V_1 - 3V_3 - 2V_2 \longrightarrow 1$

At Node 2 KCL

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$\Rightarrow 64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$ [multiplying through by 12]

$768 = 4V_1 - 4V_2 - 3V_2 - 0$

$768 = 4V_1 - 7V_2 \longrightarrow 2$

Node 3 KCL

$64 + i_1 = i_5$

$64 = i_5 - i_4$

$\Rightarrow 64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = (V_3 - 0) - 3(V_1 - V_3)$ [multiplying through by 6]

$384 = V_3 - 3V_1 + 3V_3$

$384 = -3V_1 + 4V_3 \longrightarrow 3$

$5V_1 - 2V_2 - 3V_3 = 60 \longrightarrow 1$

$4V_1 - 7V_2 + 0V_3 = 768 \longrightarrow 2$

$-3V_1 + 0V_2 + 4V_3 = 384 \longrightarrow 3$

Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta} \quad V_2 = \frac{\Delta_2}{\Delta} \quad V_3 = \frac{\Delta_3}{3}$

$$\Delta_1 = \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix}$$

$= 60(-28 - 0) - 768(-8 - 0) - 384(0 - 21)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{-3600}{-45} = 80V$

$$\Delta_2 = \begin{bmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{bmatrix}$$

$= 5(3072 - 0) - 4(240 - \frac{381}{-558}) - 3(0 - \frac{765}{-384})$

$= 2880$

$V_2 = \frac{2880}{-45} = -64V$

$$\Delta_3 = \begin{bmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{bmatrix}$$

$= 5[3072 - 0] - 4[240 - 381] - 3[0 - 765]$

$= 2880$

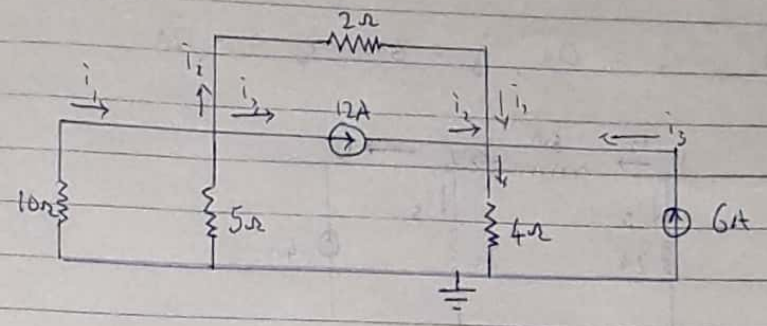
$= \frac{2880}{-45} = -64V$

$= 5(-2688 - 0) - 4(-768 - 0) - 3(-1536 - -420)$

$= -7020$

$V_3 = \frac{-7020}{-45} = 156V$

2



Node 1 KCL

$i = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_3}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \longrightarrow 1$

Node 2

$i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \longrightarrow 2$

Using elimination method

$120 = -8V_1 + 5V_2 \longrightarrow 1 \times 4$

$144 = -4V_1 + 6V_2 \longrightarrow 2 \times 8$

$-480 = 32V_1 - 20V_2 \longrightarrow 3$

$-1152 = 32V_1 - 48V_2 \longrightarrow 4$

$-672 = 0 - 28V_2$

$V_2 = \frac{672}{28} = 24V$

Substitute "$V_2 = 24$" in eqn 2

$144 = -4V_1 + 6(24)$

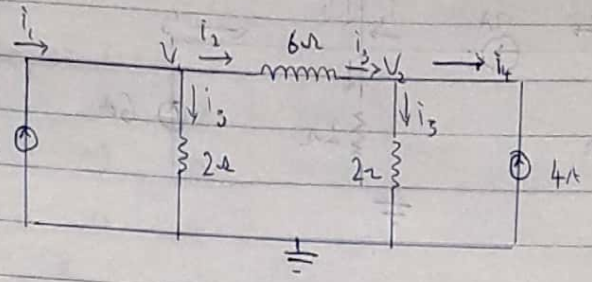
$144 = -4V_1 + 144$

$V_1 = \frac{144 - 144}{-4} = 0$

VISTALINE

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$ $i_4 = -12A$

3



Node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \longrightarrow 1$$

Node 2

$$i_2 = i_4 + i_3$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \longrightarrow 2$$

From eqn 1

$$V_2 = 4V_1 - 6$$

Substitute "$V_2 = 4V_1 - 6$" into eqn 2

$$168 = 7V_1 - 13[4V_1 - 6]$$

$$168 = 7V_1 - 52V_1 + 78$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45} = -2V$$

Substitute "$V_1 = -2$" in eqn 1

$$6 = 4[-2] - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = 14V$$

$V_1 = -2V$ & $V_2 = 14V$

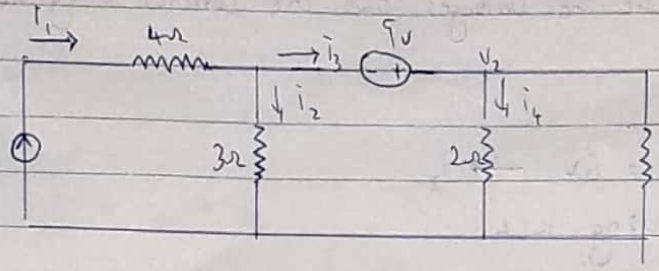
Current through the resistor

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 4}{6} = 2A$$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_3 = \frac{V_1}{7} = \frac{-14}{7} = -2A$

4



Find the current through the 3Ω & 2Ω resistor

Using KCL at Node

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0 \longrightarrow 1$

taking KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9 \longrightarrow 2$

From eqn 2

$V_2 = 9 + V_1$

Substitute "$V_2 = 9 + V_1$" in eqn 1

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = 63 - 72$

$15V_1 = -9$

$V_1 = -0.6V_1$

Substitute "$V_1 = -0.6$" in eqn 1

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

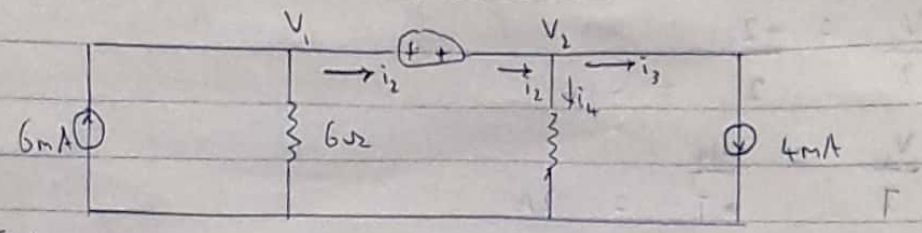
$V_2 = 9 - 0.6 = 8.4V_1$

Current through 3Ω & 2Ω

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} V = -0.2A$ at 3Ω

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$ at 2Ω

5



Find the nodes of the voltages and the currents thrgh the 6Ω and 12Ω resistor

Take $V_1 - V_2 = 6V \longrightarrow i_2$

At node 1 using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \longrightarrow 1$

At Node 2

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12[V_1 - V_2] = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2 \longrightarrow 2$

Solving V_1 & V_2 simultaneously we have

$V_1 = 9.5V$ & $V_2 = 5.1V$

Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4m$

Current through i_4

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V \qquad V_2 = 5.1V$

$i_1 = 1.58V \qquad i_4 = 0.43A$