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MECHATRONICS ENGINEERING

ENG 322.

1). At node 1, KCL:

$10 = i_1 + i_2$

$10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_2) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_2 + 2V_1 - 2V_2$

$= 5V_1 - 2V_2 - 3V_2$ --- (i)

At node 2, KCL:

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$= \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$= 4V_1 - 4V_2 - 3V_2$

$= 4V_1 - 7V_2$ --- (ii)

At node 3, KCL:

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$= \frac{V_3 - 0}{6} - \frac{V_1 - V_2}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$= -3V_1 + 4V_3$ --- (iii)

Using Cramer's Rule:

$5V_1 - 2V_2 - 3V_3 = 60 \quad \cdots (i)$

$4V_1 - 7V_2 = 768 \quad \cdots (ii)$

$-3V_1 + 4V_3 = 384 \quad \cdots (iii)$

In matrix representation:

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta} \qquad V_2 = \frac{\Delta_2}{\Delta} \qquad V_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix} = 5(-28) + 2(16) - 3(-21)$$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28) - 768(-8) - 384(-21)$$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = \underline{80V}$

$$\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix} = 5((768 \times 4) - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - (768 - 3))$$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$$V_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 708 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 708) - (-7 \times 60)]$

$= -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156\,V$

Therefore: $V_1 = 80V$; $V_2 = -54V$; $V_3 = 156V$

2) i

At node 1; KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \cdots (i)$

At node 2; $i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = 4V_1 + 6V_2 \quad \cdots (ii)$

Using Elimination method:

$120 = -8V_1 + 5V_2 \quad \times -4 \quad --$

$144 = -4V_1 + 6V_2 \quad \times -8$

$-480 = 32V_1 - 20V_2 \quad -- (iii)$

$-1152 = 32V_1 - 48V_2 \quad --- (iv)$

Subtract eqn (iii) from (iv)

$-672 = 0 - 28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24\,V$

Substitute $V_1 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6V_2}{-4}$

$V_1 = \frac{144 - 6V_2}{4}$

$V_1 = 0$

$V_1 = 0\,V \quad ; \quad V_2 = 24\,V$

~~$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$ $i_4 = 12A$~~

$i_1 = 6A \quad i_2 = 12A \quad i_3 = 6A \quad i_4 = 12A$

$i_5 = 0A \quad i_6 = 0A$

ii) At node 1:

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + V_1/2$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ --- (i)

At node 2;

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

multiplying thgh by 42:

$7V_1 - 7V_2 = 168 + 6V_2$

Collecting like terms

$7V_1 - 13V_2 = 168$ -- (ii)

$4V_1 - V_2 = 6$

$7V_1 - 13V_2 = 168$

multiplying thgl by 13.

$52V_1 - 13V_2 = 78$ -- (iii)

$7V_1 - 13V_2 = 168$ -- (iv)

Subtracting (iii) from (iv)

$45V_1 = -90$

$V_1 = -2V$

Substitute V_1 into (i)

$-8 - V_2 = 6$

$V_2 = -14$

$i_1 = 1A$

$i_2 = \frac{-2-(-14)}{2} = 12/6 = 2A$

$i_3 = \frac{-2}{2} = -1A$

$i_5 = \frac{-14}{7} = -2A$

& $i_4 = 4A$

$\therefore i_1 = 1A \quad i_2 = 2A \quad i_3 = -1A \quad i_5 = -2A \quad i_4 = 4A$