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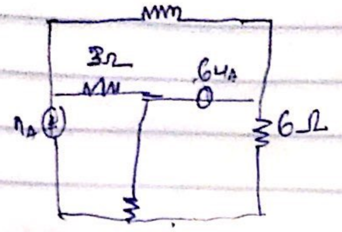
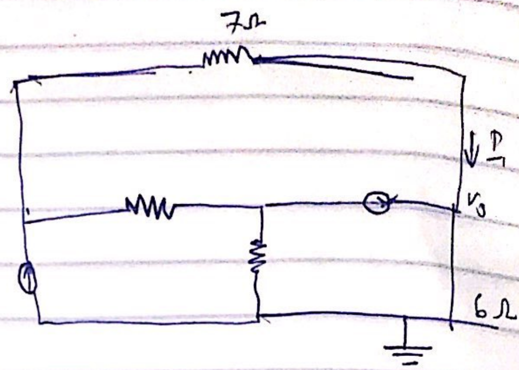
17/ENG02/068
Computer Engineering

EEE322

1. Find the voltages at nodes 1, 2 & 3 in the current below





At nodes ; KCL:

$10 = I_1 + I_2 \rightarrow 10 \quad \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$

$\rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_3$ --- (i)

At node 2, KCL

$I_2 = I_3 + 64$

$64 = I_2 - I_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3[V_2 - 0]$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2$ -- (ii)

At Node 3 KCL

$64 + I_1 = I_5$

$64 = I_3 - I_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ -- (iii)

Using Cramers Rule

$5V_1 - 2V_2 - 3V_3 = 60 \quad \ldots (i)$

$4V_1 - 7V_2 = 768 \quad \ldots (ii)$

$-3V_1 + 4V_3 = 384 \quad \ldots (iii)$

In matrix representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta} \quad V_3 = \frac{\Delta_3}{\Delta}$

when $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 6 & 4 \end{vmatrix} + -1$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) + 384(0-4)$

$= 168 + 6144 - 8064$

$= -3600$

$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5[(768+4)-0] - 4(160+4)(384-3)] - 3[0-768-3)]$

$= 7880$

$$\therefore V_2 = \frac{\Delta_1}{\Delta} = \frac{2880}{-45} = -69V$$

For V_3 :
$$\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ 3 & 0 & 384 \end{vmatrix}$$

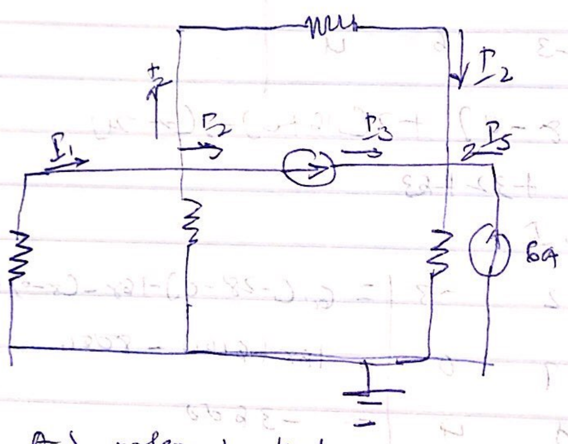
$-5[(-7 \times 384) - 0] - 4((-2 \times 384 - 0) - 3(-2 \times 768) - (-7 \times 60)]$

$= -7020$

$$\therefore V = \frac{\Delta_1}{\Delta} = \frac{-7020}{-45} = 156V$$

Hence $V_1 = 80V$, V_1 $-69V$ $V_3 = 156V$

2 * Find the voltage at node 1 & 2 and determine the current flowing through the resistors in the circuit below



At nodes 1 KCL

$I_1 = I_2 + I_3 + I_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 20 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ --- (1)

Scanned with CamScanner

At node 2

$I_1 + I_2, I_3 = I_4$

$$V_2 = \frac{V_1 - V_2}{2} = 6 = \frac{V_2 - 0}{4}$$

$16 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \text{--- (ii)}$

Using elimination method

$120 = -8V_1 + 5V_2 \quad \text{--- (i)} \times -4$

$144 = -4V_1 + 6V_2 \quad \text{--- (ii)} \times -8$

$-480 = 32V_1 - 20V_2 \quad \text{--- (iii)}$

$-1152 = 32V_1 - 48V_2 \quad \text{--- (iv)}$

sub eqn (iii) to eqn (iv)

$-671 = 0\,28V$

$$V_2 = \frac{-672}{-28} \therefore V_2 = 24V$$

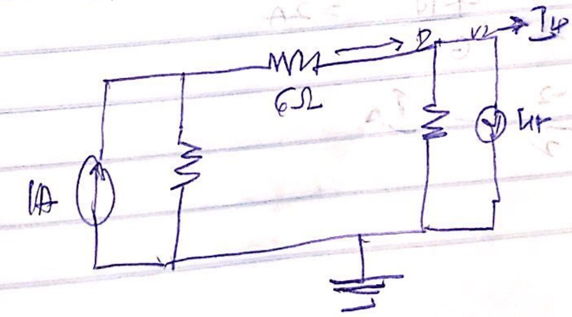
~~sub~~

$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}, \quad V_1 = \frac{144 - 6V_2}{-4} \therefore V_1 = 0$$

$\therefore V_1 = 0V, \; V_2 = 24V, \; I_1 = 0A \quad I_2 = 6A \; I_3 = 6A \; I_4 = -12A$

(ii) Often V_1 & V_2 are the current through the resisting of the circuit in example (ii) if the ~~3A~~ current source was replaced by 1A current source



At node 1

$I_1 = I_2 + I_3$

$I = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V$

$6 = 4V_1 - V_2$ -- (i)

At node 2

$I_2 = I_4 + I_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ -- (ii)

from equ (i) $V_2 = 4V_1 - 6$

Sub $(4V_1 - 6)$ for V_2 in equ (ii)

$168 = 7V_2 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V$

$V = \frac{90}{-45}$

$V_1 = -2V$

Sub. -2 for V_1 in equ

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_1 = -2V$, $V_2 = -14V$

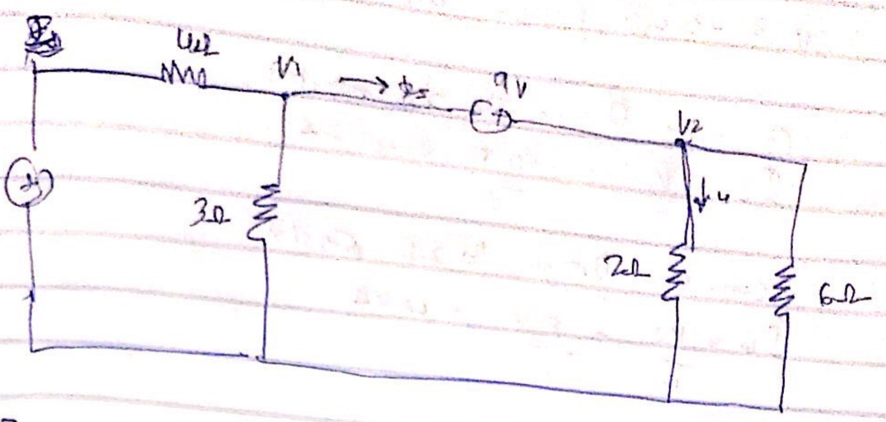
current through the resistor

$I_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$I_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$I_s = \frac{V_2}{4} = \frac{-16}{7} = -2A$

(3)



first the current through the 3Ω and 2Ω resistor

Using KCL at node 1

$I_1 + I_2 + I_3 + I_4 = 0$

$\frac{V_1 - 21}{4} - \frac{V_1}{3} + \frac{V_1}{6} + \frac{V_2}{6}$

$7V_1 + 8V_2 + 0 = 0$ -- (1)

Using KVL for Loop 1

$-V_1 - 9 + V_2 = 0, \; -V_1 + V_2 - 9$ -- (11)

$7V_1 + 8V_2 = 63$ -- (1)

$-V_1 + V_2 - 9$ ---- (11)

(11) V_1 $V_2 : 9 + V_1$

Sub $V_2 = 9 + V_2$ in eqn (1)

$7V_0 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V = -0.6V$

Sub $V_1 - 0.6$ in eqn (11)

$-(-0.6) + V_2 - 9$

$0.6 + V_2 - 9$

$V_2 = 5.4 V$

$\therefore V_1 = -0.6V$ & $V_2 = 5.4 V$

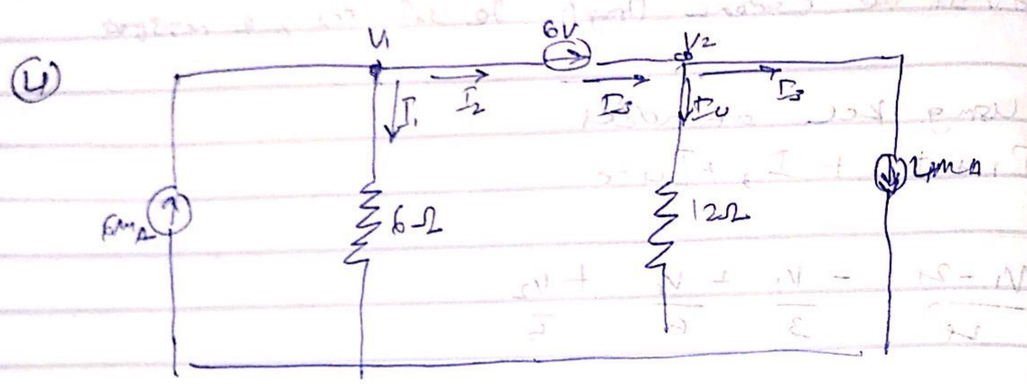
Current through the 3Ω resistor

$$I_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2Ω resistor

$$I_4 = \frac{V_2}{4} = \frac{5.4}{4} = 1.2A$$

(4)



find the node voltage and the current through the 6Ω & 12Ω resistor (lets assume that $V_1 - V_2 = 6V \rightarrow$ 2

At node I using KCL

$6mA = I_1 - I_2$

$$6mA = \frac{V_1 - 0}{6} \cdot (V_1 - V_2)$$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ —(1)

At Node 2

$I_2 = I_3 + I_4$

$$V_1 - V_2 = 4mA + \frac{V_3 - 0}{12}$$

$12(v_1 - v_2) = 48 + v_2$

$48 = 12v_1 - v_2 - v_2$

$48 = 12v - 13v_2$ - - (1)

Solving v_1 & v_2 simultaneously, we have

$v_1 = 9.5v$ & $v_2 = 5.1v$

∴ current through the 6Ω resistor

$I_5 = \frac{v_1}{6} = \frac{9.5}{6} = 1.58A$ $I_2 = 4$ $v_2 = 45 - 5$

$I_2 = 44A$

current through the 12Ω resistor

$I_4 = \frac{v_2}{12} = \frac{5.1}{12} = 0.43A$

$v_1 = -9.5v$; $v_2 = 5.1v$,

$I_1 = 158A$. $I_2 = 0.43A$