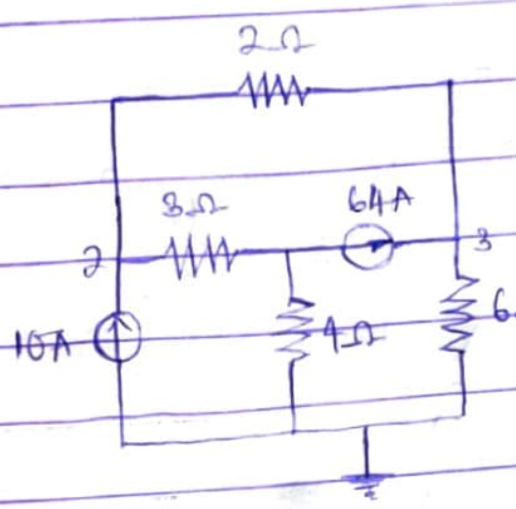
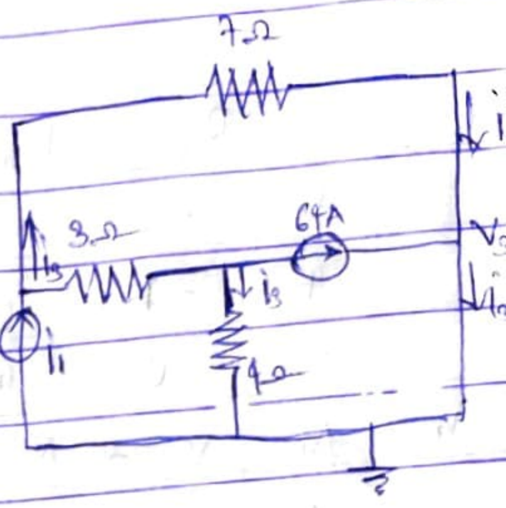
ENG 322 Assignment

1. Find the voltages at nodes 1, 2 and 3 in the circuit below





At nodes 1, KCL;

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \text{---} (i)$$

At Node 2, KCL;

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \quad \text{---} (ii)$$

At node 3, KCL;

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ --- (iii)

Using Crammer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ --- (i)

$4V_1 - 7V_2 = 768$ --- (ii)

$-3V_1 + 4V_3 = 384$ --- (iii)

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-280) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For V_2 : $\Delta_2 = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 \cdot 3)) - 3(0 - (768 \cdot 3))$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V.$

For V_3;

$$\begin{array}{c|ccc|} + & 5 & -2 & 60 \\ - & 4 & -7 & 768 \\ + & -3 & 0 & 389 \end{array}$$

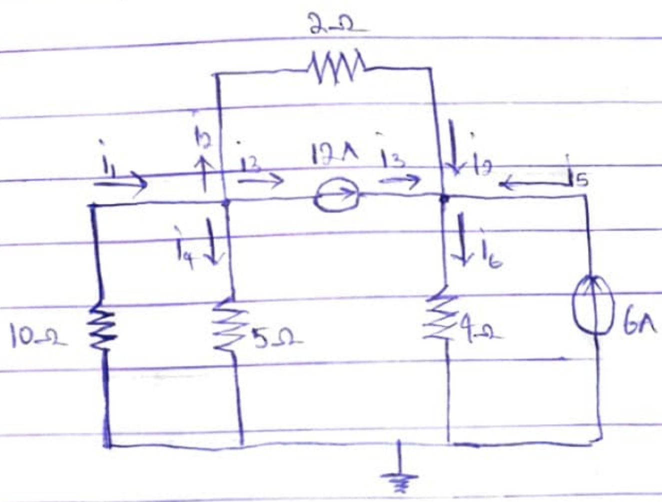
$= 5\big((-7 \times 389) - 0\big) - 4\big((-2 \times 389 - 0) - 3\big((-2 \times 768) - (-7 \times 60)\big)$

$= -7020$

$\therefore V_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{-7020}{-45} = 156V$

Hence $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

i Find the voltages at node 1 and 2 and determine the Currents flowin through the four resistors in the Circuit below



At Node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$\dfrac{V_2 - V_1}{10} = \dfrac{V_1 - V_2}{2} + 12 + \dfrac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ --- (i)

At nodes 2

$i_3 + i_2 + i_5 = i_6$

$12 + \dfrac{V_1 - V_2}{2} + 6 = \dfrac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 = -4V_1 + 6V_2$ --- (ii)

Using elimination method

$120 = -8V_1 + 5V_2$ --- (i) $\times 4$

$144 = -4V_1 + 6V_2$ --- (ii) $\times 8$

$-480 = 32V_1 - 20V_2$ --- (iii)

$-1152 = 32V_1 - 48V_2$ --- (iv)

Subtract eqn (iii) from (iv)

$-672 - 0 = -28V_2$

$$V_2 = \frac{-672}{-28}$$

$V_2 = 24V$

Subs $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}$$

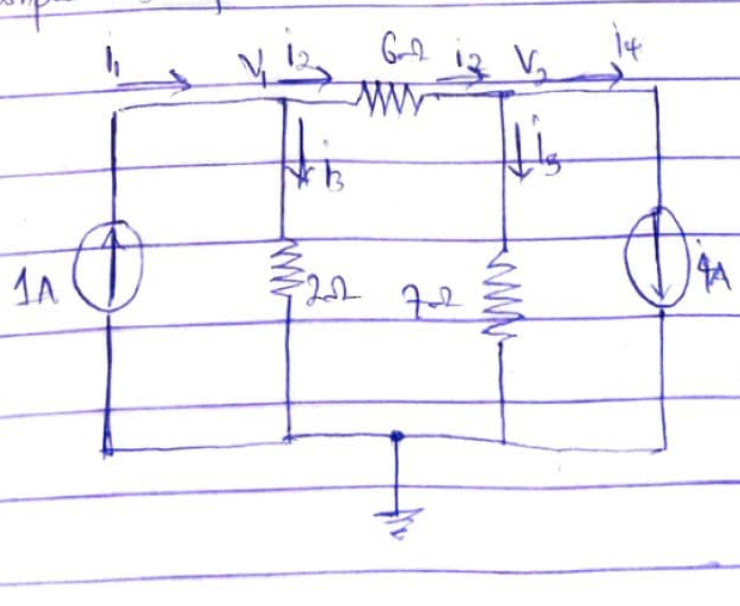
$$V_1 = \frac{144 - 6V_2}{-4}$$

$V_1 = 0$

$\therefore V_1 = 0V$, $V_2 = 24V$

$i_1 = 0A$, $i_2 = 0A$, $I_3 = 6A$, $i_4 = 12A$.

Obtain V_1 and V_2 and the currents through the resistors for the circu example (ii) if the 2A current source was replaced by a 1A curre



At Node 1

$i_1 = i_2 + i_3$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_2}{2}$$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ --- (i)

At Node 2

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ --- (ii)

From eqn (i); $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V.$

Subs $V_1 = -2$ in eqn (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$-V_2 = 6+8$

$-V_2 = 14$

$V_2 = -14$

$V_1 = -2V;\ V_2 = -14V$

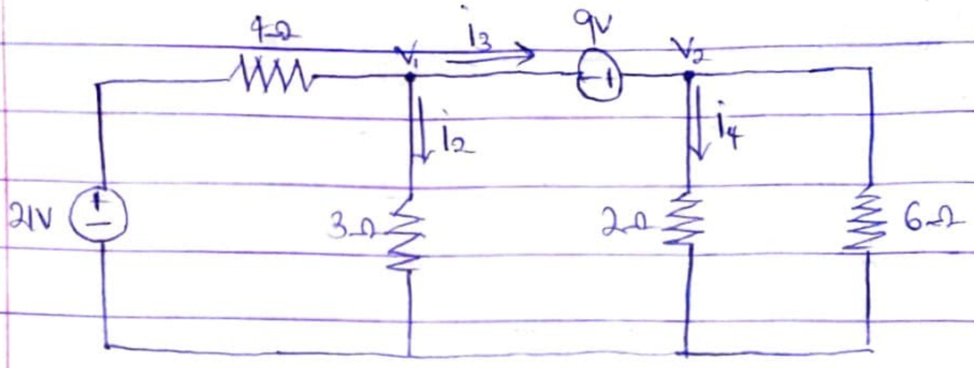
Current Through the resistors

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$

(3)



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \text{---(i)}$$

Using KVL for loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \text{---(ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{---(i)}$$

$$-V_1 + V_2 = 9 \quad \text{---(ii)}$$

from (ii) let $V_2 = 9 + V_1$

Substitute $V_2 = 9 + V_1$ in eqn (i)

$$7V_1 + 8(9 + V_1) = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = -9$$

$$V_1 = -0.6V$$

Substitute $V_1 = -0.6$ in eqn (ii)

$$-(-0.6) + V_2 = 9$$

$$0.6 + V_2 = 9$$

$$V_2 = 8.4V$$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$.

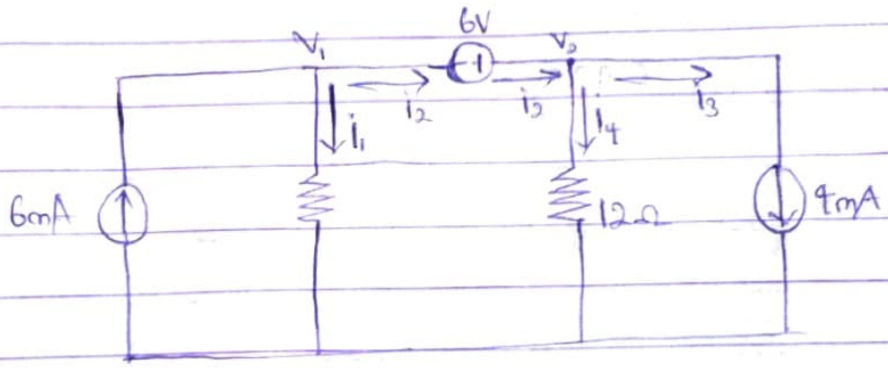
Current through the 3Ω resistor

$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2Ω resistor

$$i_4 = \frac{V_2}{2} = \frac{8.4}{2} = 4.2A$$

(4)



Find the node voltages and the currents through the 6Ω and 12Ω resistors

Lets assume that $V_1 - V_2 = 6V \rightarrow i_2$

At node 1, using KCL

$6mA = i_1 + i_2$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ ---(i)

At node 2;

$i_2 = i_3 + i_4$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2$ --- (ii)

Solving V_1 and V_2 simultaneously, we have $V_1 = 9.5V$ and $V_2 = 5.1V$.

∴ Current through the 6Ω resistor

$i_1 = V_1/6 = 9.5/6 = 1.58A$; $i_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

∴ $V_1 = 9.5V$; $V_2 = 5.1V$

$i_1 = 1.58A$; $i_4 = 0.43A$