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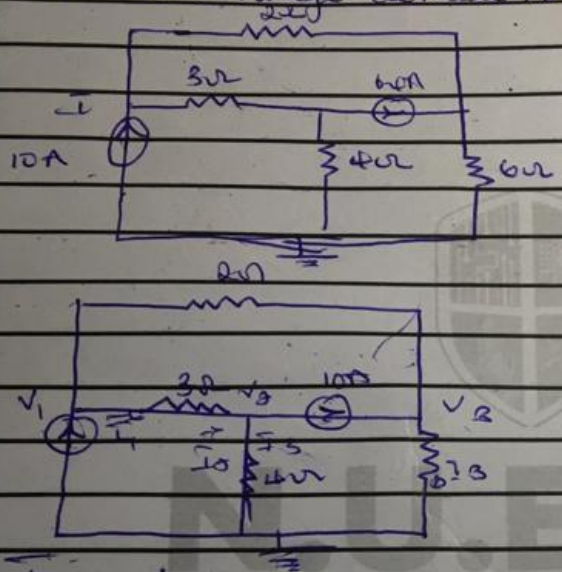
17/ENG02/009
Computer Engineering
EEE 8

Assignment:

Find the voltage at node 1, 2, and 3 in the circuit below



From Node 1

$10 = i_1 + i_2 \rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

Multiplying through by 6

$60 = 3[V_1 - V_3] + 2[V_1 - V_2]$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 3V_3 - 2V_2$ .. (1)

From node 2,

$i_2 = i_3 + 6i_4$

$6i_4 = i_2 - i_3$

$6i_4 = V_1 - V_2/3 - V_2 - 0/4$

Multiply through by 4

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 7V_2$ --- (2)

From node 3

$6A + 2A = 15$

$6A = 15 - 2A$

$6A = \frac{V_3 - 0}{6} - \frac{V_2 - V_3}{2}$

$36A = V_3 - 3(V_1 - V_3)$

$36A = -3V_1 + 4V_3$ --- (3)

$5V_1 - 2V_2 - 3V_3 = 60$ --- (1)

$5V_1 - 7V_2 + 0V_3 = 768$ --- (2)

$-3V_1 + 0V_2 + 4V_3 = 36A$ --- (3)

In Matrix form

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 36A \end{bmatrix}$$

$V_1 = \Delta_1 / \Delta$; $V_2 = \Delta_2 / \Delta$; $V_3 = \Delta_3 / \Delta$

$$\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

$= 5[-28 - 0] + 2[16 + 0] - 3[0 - 21]$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix}$$

$= 60[-20-0] - 768[-8-0] - 384[0-21]$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \Delta_1 / \Delta = -3600 / -45$

$= 80V$

$$V_2 = \Delta_2 / \Delta = \begin{bmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{bmatrix}$$

$= 5([768 \times 4) - 0] - 4[60 \times 4] - [384 - 3)] - 3[0 - [768 \times -3]]$

$= 2880$

$V_2 = 2880 / -45 = -64V$

$$V_3 = \begin{bmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{bmatrix}$$

$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 768) - (-7 \times 60)]$

$= 7020$

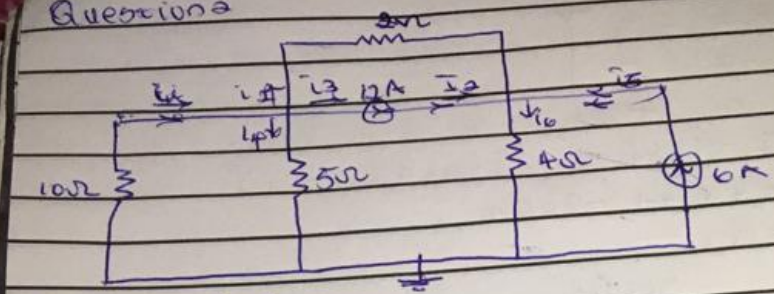
$V_3 = \Delta_3 / \Delta = -7020 / -45 = 156V$

$V_1 = 80V$

$V_2 = -64V$

$V_3 = 156V$

Question



From node 1:

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_3}{2} + 12 + \frac{V_1 - V_2}{5}$

$\Rightarrow -V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$\Rightarrow V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad -- (1)$

From Node 2:

$i_3 + i_4 + i_5 = 16$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$48 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad -- (2)$

$120 = -8V_1 + 5V_2 \quad -- (1) \quad -- \times 4$

$144 = -4V_1 + 6V_2 \quad -- (2) \quad -- \times 8$

$-480 = 32V_1 - 20V_2$

$-1152 = 32V_1 - 48V_2$

$-672 = 28V_2$

$V_2 = -672/-28$

$V_2 = 24V$

Equating ($V_2 = 24V$) into eqn(1)

$144 = -4V_1 + 6V_2$

$V_1 \Rightarrow 144 - 6(24)/-4$

$V_1 = 144 - 144/4$

$\Rightarrow 0/4$

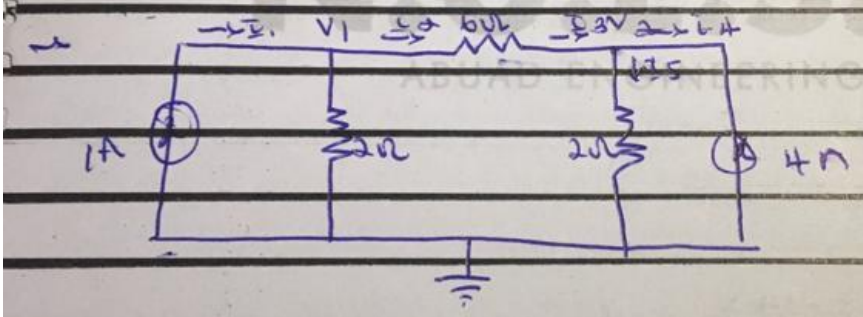
$V_1 = 0$

$V_1 = 0V$;

$V_2 = 24V$

$i_1 = 0A \quad i_3 = 6A$

$i_2 = 0A \quad i_4 = -12A$



From Node 1

$i_1 = i_2 = i_3$

$\frac{V_1 - V_2}{6} + \frac{V_1}{6}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \quad (1)$

$V_2 = 4 + 10$

$\frac{V_1 - V_2}{6} = \frac{4 + V_2}{7}$

$7[V_1 - V_2] = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \quad \ldots (1)$

From equation 1

$V_2 = 4V_1 - 6$

Equating ($V_2 = 4V_1 - 6$ into eqn (1))

$168 = 7V_1 - 13(4V - 6)$

$168 = 7V_1 - 53V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Substituting $V_1 = 0$ into eqn (1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

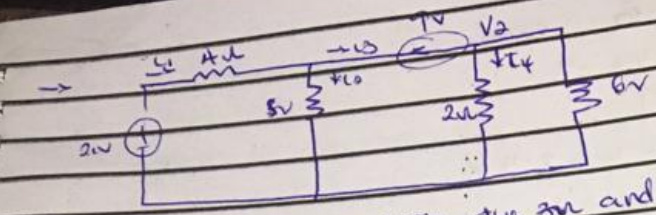
$V_1 = -2V \; ; V_2 = -14V$

Calculating current through the resistor

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{6} = \frac{-2}{2} = -1A$

$i = \frac{V_2}{4} = \frac{-14}{4} = -2A$



Find the current through the 3Ω and 2Ω resistor

Using KCL at Node1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 2_1}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \ldots (1)$$

taking KVL for loop1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \ldots$$

$$7V_1 + 8V_2 = 63 \quad \ldots x1$$

$$-V_1 + V_2 = 9 \quad \ldots 7$$

$$7V_1 + 8V_2 = 63$$

$$-7V_1 + 7V_2 = 63$$

$$11V_2 = 126$$

From equation 2 : $-V_1 + V_2 = 9$

$$V_2 = 9 + V_1$$

Substituting into equation 1

$$7V_1 + 8[9 + V_1] = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = 63 - 72$$

$$15V_1 = -9$$

$$V_1 = -9/15$$

$$V_1 = -0.6V$$

Equating $V_1 = 0.6$ in equation (1)

$-[-0.6] + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 9 - 0.6 = 8.4V$

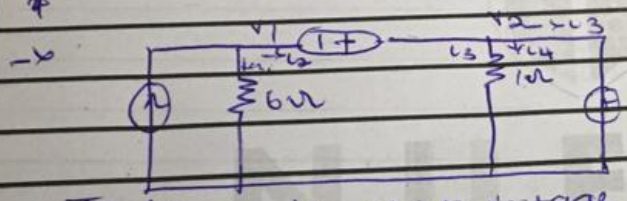
$V_1 = -0.6V$

$V_2 = 8.4V$

Calculating current through 3Ω and 2Ω

$i_3 = -V_1/3 = -0.6/3 = 0.2A$ at 3Ω

$i_4 = V_2/4 = 8.4/4 = 4.2A$ at 2Ω



Find the nodes at the voltage and the current through 6Ω and 12Ω resistor

From node 1,

Take $V_1 - V_2 = 6V = V_a$

$6mA = i_1 + i_2$

$6mA = (V_1 - 0)/6 + (V_1 - V_2)/1$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad -(1)$

From Node 2

$i_2 = 18 + i_4$

$V_1 - V_2 = 4mA + V_2 - 0 / 12$

$12[V_1 - V_2] = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$ ∴

$48 = 12V_1 - 13V_2$ --(ii)

Solving V_1 and V_2 simultaneously

$V_1 = 9.5V$ and $V_2 = 5.1V$

Calculating current through the 6Ω resistor

$i_1 = V_1 / 6 = 9.5 / 6 = 1.58A$

$V_3 = V_1 - V_2 = 9.5 - 5.1 = 4.4mV$

Current through 12Ω

$I_4 = V_2 / I_1 = 5.1 / 12 = 0.43A$

$V_1 = 9.5V$

$V_2 = 5.1V$

$I_1 = 1.58V$

$I_2 = 0.43A$