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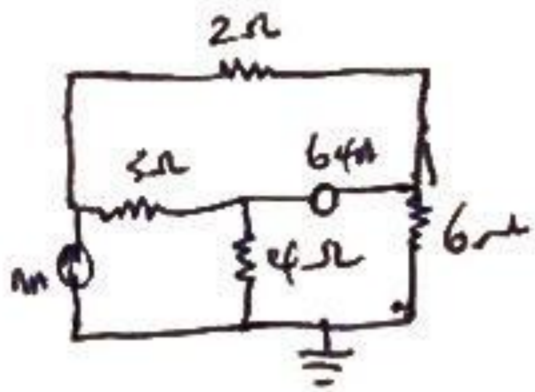
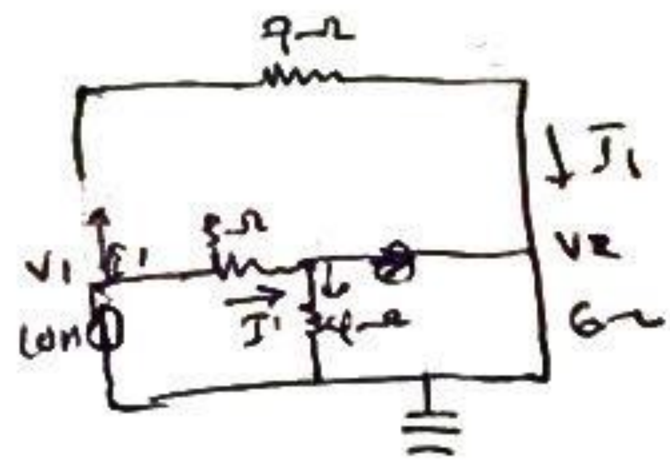
17/5C01/047

Computer Engineering

ECE 322

1] Find the voltage at node 1, 2 & 3 in the circuit below.





At node 1, KCL,

$10 = I_1 + I_2 \rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3}$

$\rightarrow 60 = 3(V_1 - V_2) + 2(V_1 - V_3)$

$60 = 3V_1 - 3V_2 + 2V_1 - 2V_3$

$60 = 5V_1 - 2V_2 - 3V_3$ —— (1)

At node 2 KCL

$I_2 = I_3 + 64$

$I_2 = I_2 - I_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3[V_2 - 0]$

$= 4V_1 - \cancel{4}V_2 - 3V_2$

$768 = 4V_1 - 7V_2$ — (2)

At Node 3 KCL

$64 + I_1 = I_5$

$64 / = A$

P.T.O

$64 = I_5 - I_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \quad \cdots\cdots (iii)$

Using Cramer Rule

$5V_1 - 2V_2 - 3V_3 = 60 \quad \cdots\cdots (i)$

$4V_1 - 7V_2 = 768 \quad \cdots\cdots (ii)$

$-3V_1 + 4V_3 = 384 \quad \cdots\cdots (iii)$

I. Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$

$$\begin{array}{cccc} & + & - & + \\ \text{when } \Delta = & \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix} & & \end{array} \; + - +$$

$= 5(-28 - 0) + 2(16 + 0) - 3(0 - 21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} \begin{aligned} &= 60(-28 - 0) - 768 \cdot (-8 - 0) + 384(0 - 21) \\ &= -168 + 6144 - 8064 \\ &= -3600 \end{aligned}$$

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$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$V_2 : \Delta_2 = \begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5[[(768 \times 4) - 0] - 4(160 \times 4) - (384 - 3)] - 3(0 - 768 - 3)]$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

for V_3 ; $\begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

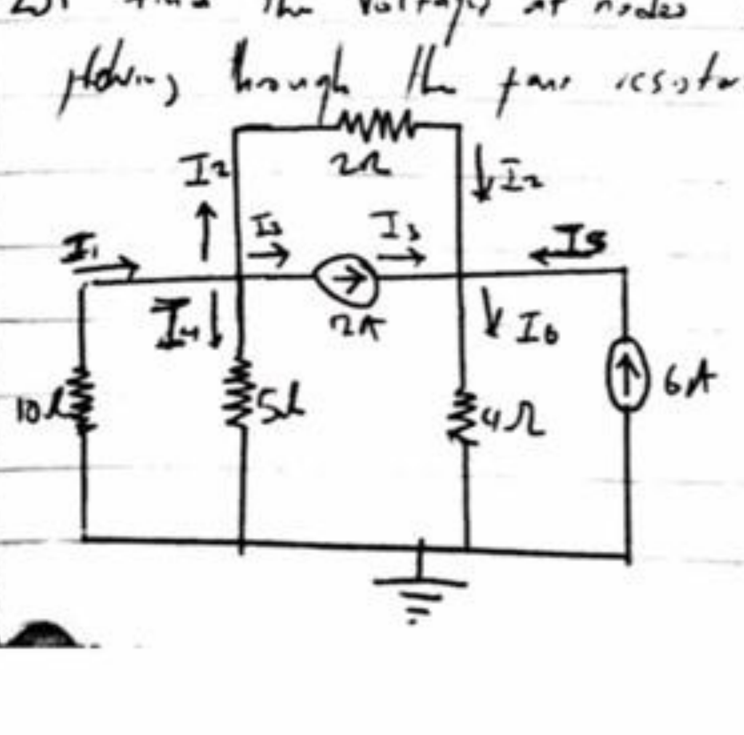
$= 5[[(-7 \times 384) - 0) - 4((-2 \times 384 - 0) - 3((-2 \times 768) - (-7 \times 60)]$

$= -7020$

$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

Hence $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

2). Find the Voltages at nodes 1 & 2 and determine the current flowing through the four resistors in the circuit below



At node 1, KCL

$I_1 = I_2 + I_3 + I_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \cdots (1)$

At node 2

$I_1 + I_2 + I_3 = I_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \cdots (11)$

Using elimination method

$120 = -8V_1 + 5V_2 \cdots (1) \times -4$

$144 = -4V_1 + 6V_2 \cdots (11) \times -8$

$-480 = 32V_1 - 20V_2 \cdots (111)$

$-1152 = 32V_1 - 48V_2 \cdots (1V)$

Sub eqn (111) from (1V)

$-672 = 0 - 28V_2$

$$V_2 = \frac{-672}{-28} \therefore V_2 = 24V$$

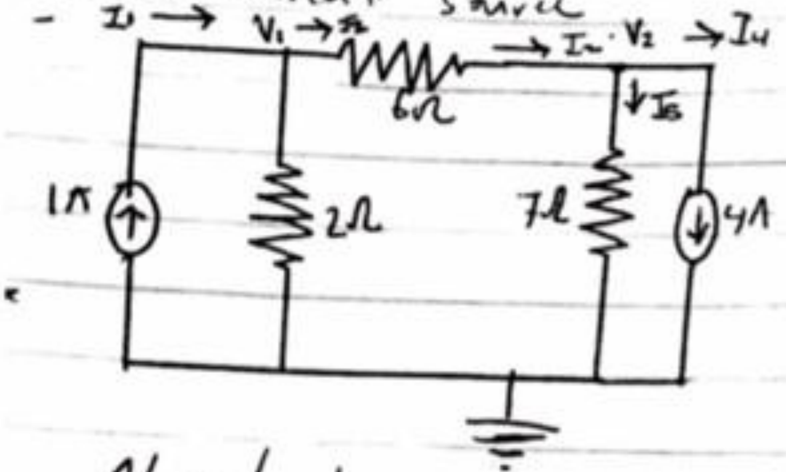
Sub ~~that~~ for V_2 in eqn (11)

$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}, \quad V_1 = \frac{144 - 6V_2}{-4} \therefore V_1 = 0$$

$\therefore V_1 = 0V, \; V_2 = 24V, \; I_1 = 0A, \; I_2 = 0A, \; I_3 = 6A, \; I_4 = -12A$

ii] obtain V_1 & V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current source



At node 1

$I_1 = I_2 + I_3$

$$I = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \ldots (i)$

At Node 2

$I_2 = I_4 + I_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \quad \ldots (ii)$

from eqn (i): $V_2 = 4V_1 - 6$

Sub $(4V_1 - 6)$ for V_2 in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = \frac{90}{-45}$

$V_1 = -2\ V$

Sub -2 for V_1 in eqn (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_1 = -14V$

$\therefore V_1 = -2V, \quad V_2 = -14V$

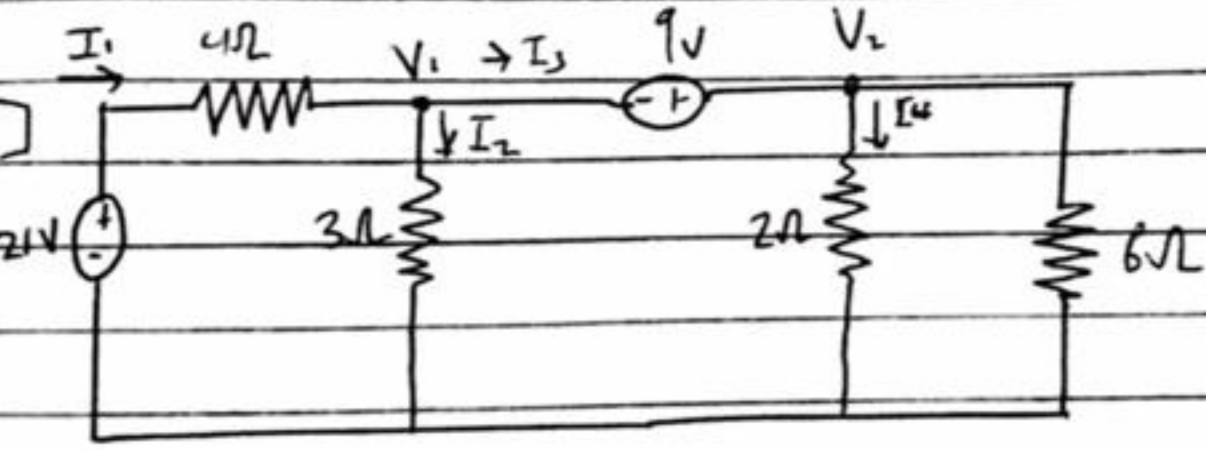
Current through the resistors

$I_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$I_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$I_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

3]



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$I_1 + I_2 + I_3 + I_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0 \quad \cdots (i)$

Using KVL for loop 1

$-V_1 - 9 + V_2 = 0, \quad -V_1 + V_2 = 9 \; --- (ii)$

$7V_1 + 8V_2 = 63 \quad \ldots\ldots (I)$
$-V_1 + V_2 = 9 \quad \ldots\ldots (II)$

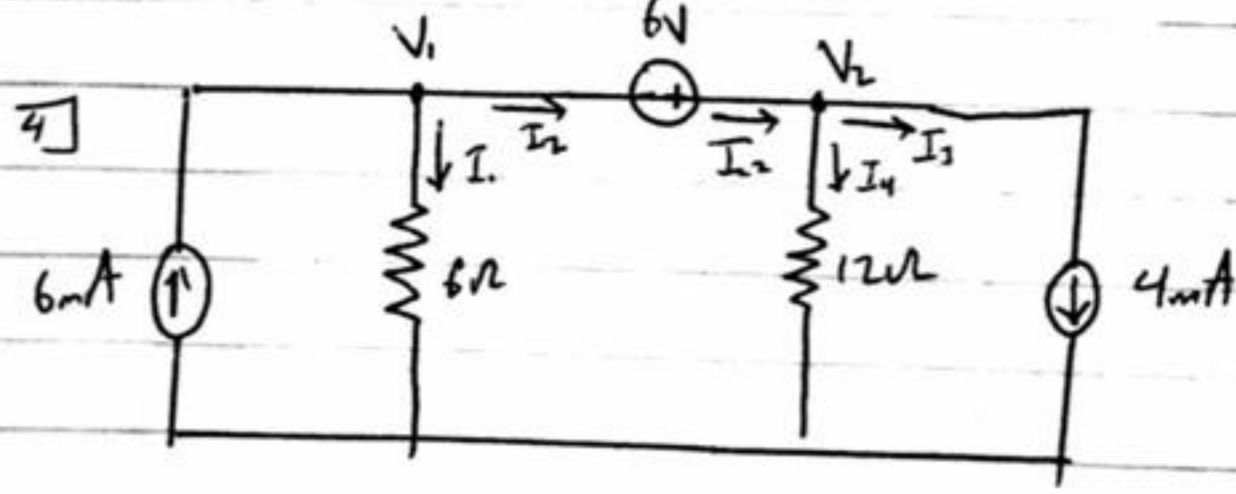
fm (II) let $V_2 = 9 + V_1$
Sub $V_2 = 9 + V_1$ in eqn (I)
$7V_1 + 8(9 + V_1) = 63$
$7V_1 + 72 + 8V_1 = 63$
$15V_1 = -9$
$V_1 = -0.6V$

Sub $V_1 = -0.6$ in eqn (II)
$-(-0.6) + V_2 = 9$
$0.6 + V_2 = 9$
$V_2 = 8.4$
$\therefore V_1 = -0.6V$ & $V_2 = 8.4V$

Current through the 3Ω resistor
$I_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor,
$I_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4]



Find the node voltage and the current through the 6Ω & 12Ω resistor
(Let's assume that $V_1 - V_2 = 6V \rightarrow I_2$

At node I, using KCL

$6mA = I_1 + I_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad \cdots (i)$

At Node 2

$I_2 = I_3 + I_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \quad \cdots (ii)$

Solving V_1 & V_2 simultaneously, we have

$V_1 = 9.5V$ & $V_2 = 5.1V$

$\therefore$ Current through the 6Ω resistor

$I_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$; $I_2 = V_1 - V_2 = 9.5 - 5.1$

$I_2 = 4.4A$

Current through the 12Ω resistor

$I_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore V_1 = 9.5V$; $V_2 = 5.1V$

$I_1 = 1.58A$; $I_4 = 0.43A$

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