Circuit theory

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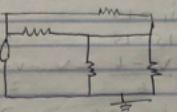
① Find the voltage at node 1, 2 and 3 in the circuit below



Node 1 KCL

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

multiply through by 6

$$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$
$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$
$$60 = 5V_1 - 3V_3 - 2V_2 \quad -0$$

At node 2 KCL

$$I_2 = i_3 + 6u$$
$$6u = i_2 - i_3$$
$$6 + = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

Multiply through by 7

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 7V_2 \quad -②$

Node 3 KCL

$64 + i_1 = 13$

$64 = 15 - 14$

$64 = \frac{V_3 - 6}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3 \quad -③$

$5V_1 - 2V_2 - 3V_3 = 60 \quad -①$

$4V_1 - 7V_2 + 6V_1 = 768 \quad -②$

$-3V_1 + 0V_2 + 4V_3 = 384$

matrix eq

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \Delta_1 / \Delta \quad V_2 = \Delta_2 / \Delta_2 \quad V_3 = \Delta_3 / \Delta_3$

$$\Delta = \begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$$\Delta_1 = \begin{bmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{bmatrix}$$

$= 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 = 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$V_2 = \Delta_{22}$

$$\begin{bmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{bmatrix}$$

$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 \times -3)) - 3(0 - (768 \times -3)) = 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

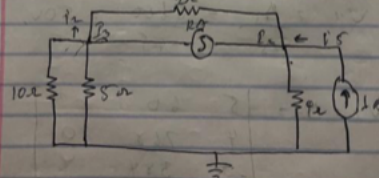
$$V_3 \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5[(-7 \times 384) - 0] - 4[(-2 \times 384) - 0] - 3[(-2 \times 768) - (-7 \times 60)] = -7020$

$V_3 = \frac{\Delta_2}{\Delta} = \frac{-7020}{-45} = 156V$

$\therefore V_1 = 80V, V_2 = -64V, V_3 = 156V$

(21)



Node 1 KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$+20 = -8V_1 + 5V_2 \quad -(i)$

Node 2

$i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 + 48 = 2V_2$

$144 = -4V_1 + 6V_2 \quad -(ii)$

Using Elimination method

$120 = -8V_1 + 5V_2 \quad -(i) \times 4$

$144 = -4V_1 + 6V_2 \quad -(ii) \times 8$

$-480 = 32V_1 + 20V_2 \quad -(iii)$

$-1152 = 32V_1 - 48V_2 \quad -(iv)$

Subtract eqn (3) from (4)

$-672 = 0 - 28V_2$

$-672 = -28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24V$

Sub V_2 into eq (i)

$144 = -4V_1 + 6V_2$

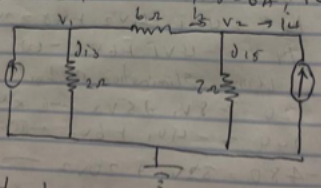
$V_1 = \frac{144 - 6(-24)}{-4}$

$V_1 = \frac{-144 - 144}{-4} = 0V$

$V_1 = 0V \cdot V_2 = 24V$

$i_1 = 0A, i_2 = 0A, i_3 = 6A, i_4 = -4A$

(ii)



Node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 2V_1$

$6 = 4V_1 - V_2$ — ①

Node 2

$I_2 = I_4 + I_3$

$\frac{V_1 - V_2}{6} = \frac{4 + V_2}{7}$

$7(V_1 - V_2) = 6(4 + V_2)$

$7V_1 - 7V_2 = 24 + 6V_2$

$\therefore 24 = 7V_1 - 13V_2$ — ②

from equation ①

$V_2 = 4V_1 - 6$

Sub $V_2 = 4V_1 - 6$ into eqn ②

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = 45V_1$

$V_1 = 90/-49$

$V_1 = -2V$

Sub $V_1 = -2$ into eqn (1)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$V_1 = -2V_S$, $V_2 = -14V$

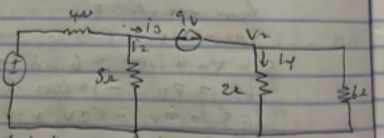
Current through the resistors

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

(3)



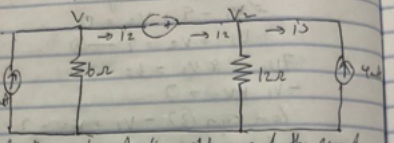
Find the current through the 3Ω and 2Ω resistors.

Use KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$i_4 = V_2/4 = 0.4/4 = 4.2A$ at 2R



Find the nodes at the voltage and the current through the 6Ω and 12Ω resistor.

Take $V_1 - V_2 = 6V \rightarrow i_2$

at node 1 using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ —①

At node 3

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2}{12} - 0$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 - (2)$

Solving V_1 and V_2 simultaneously we have

$V_1 = 9.5V$

$V_2 = 5.1V$

current through the 6Ω resistor

$i_1 = V_1/6 = 5.5/6 = 1.58A$

$I_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4mA$

Current through 12Ω

$i_4 = V_2/i_2 = 5.1/12 = 0.43A$

$V_1 = 9.5V$ $\quad V_2 = 5.1V$

$i_1 = 1.58A$ $\quad i_4 = 0.43A$