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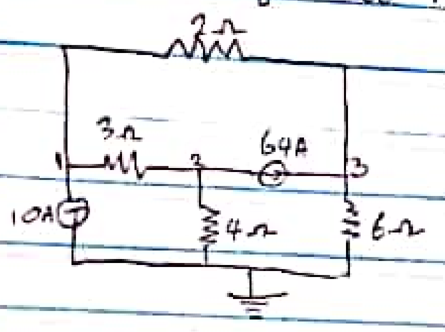
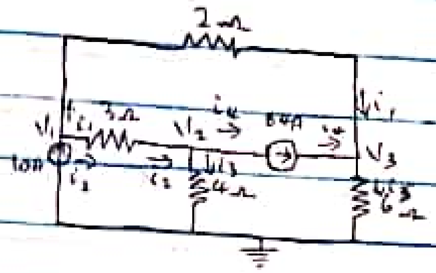
17/Eng04/068
Electrical/Electronics

1) Find the voltages at nodes 1, 2 and 3 in the circuit below





At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3}$$

$$60 = 3(v_1 - v_3) + 2(v_1 - v_2)$$

$$60 = 3v_1 - 3v_3 + 2v_1 - 2v_2$$

$$60 = 5v_1 - 2v_2 - 3v_2 \quad \text{---(i)}$$

At node 2, KCL;

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{v_1 - v_2}{3} - \frac{v_2 - 0}{4}$$

$$768 = 4(v_1 - v_2) - 3(v_2 - 0)$$

$$768 = 4v_1 - 4v_2 - 3v_2$$

$$768 = 4v_1 - 7v_2 \quad \text{---(ii)}$$

At node 3, KCL

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{v_3 - 0}{6} - \frac{v_1 - v_3}{2}$$

$$384 = v_3 - 3(v_1 - v_3)$$

$$384 = -3v_1 + 4v_3 \quad \text{---(iii)}$$

$= 5[(768\times4)-0] - 4[(60\times4)-(384\times-3)] - 3[0-(768\times-3)]$

$= 2880$

$\therefore v_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{2880}{-45} = -64V$

For v_3,
$$\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

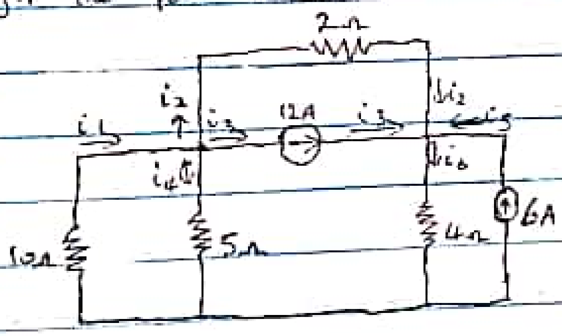
$= 5[(-7\times384)-0] - 4[(-2\times384)-0] - 3[(-2\times768)-(-7\times60)]$

$= -7020$

$v_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{-7020}{-45} = 156V$

Hence $v_1 = 80V$, $v_2 = -64V$, $v_3 = 156V$

2) i) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below



At node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$$\frac{v_0 - v_1}{10} = \frac{v_1 - v_2}{2} + 12 + \frac{v_1 - v_0}{5}$$

$0 - v_1 = 5(v_1 - v_2) + 120 + 2(v_1 - 0)$

$-v_1 = 5v_1 - 5v_2 + 120 + 2v_1$

$120 = -8v_1 + 5v_2$ ---(i)

$6 = -8 - v_2$

$v_2 = -8 - 6$

$v_2 = -14v$

$\therefore v_1 = -2v, \; v_2 = -14v$

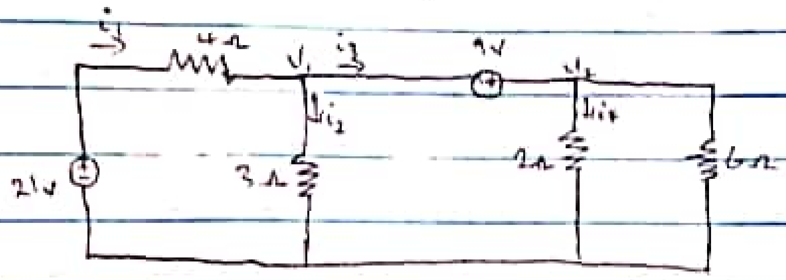
Current through the resistors

$i_2 = \frac{v_1 - v_2}{6} = \frac{-2+14}{6} = 2A$

$i_3 = \frac{v_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{v_2}{7} = \frac{-14}{7} = -2A$

3)



Find the current through the 3Ω and 2Ω resistors

Using kcl at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{v_1 - 21}{4} + \frac{v_1}{3} + \frac{v_2}{6} + \frac{v_2}{2}$

$7v_1 + 8v_2 - 63 = 0$ ---(i)

Using kvl for loop 1

$-v_1 - 9 + v_2 = 0$

$-v_1 + v_2 = 9$ ---(ii)

$7v_1 + 8v_2 = 63$ ---(i)

$-v_1 + v_2 = 9$ ---(ii)

$\therefore$ Let $v_2 = 9 + v_1$

Substitute $v_2 = 9 + v_1$ in eqn(1)

At node 2

$i_2 = i_3 + i_4$

$v_1 - v_2 = 4mA + \frac{V_2 - 0}{12}$

$12(v_1 - v_2) = 48 + v_2$

$48 = 12v_1 - 12v_2 - v_2$

$48 = 12v - 13v_2$ --- (1)

Solving v_1 and v_2 simultaneously, we have

$v_1 = 9.5V$ and $v_2 = 5.1V$

Current through the 6Ω resistor

$i_1 = \frac{v_1}{6} = \frac{9.5}{6} = 1.58A$, $i_2 = v_1 - v_2 = 9.5 - 5.1 \Rightarrow 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore v_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_4 = 0.43A$

$$7v_1 + 8(9+v_1) = 63$$

$$7v_1 + 72 + 8v_1 = 63$$

$$15v_1 = -9$$

$$v_1 = -0.6V$$

Substitute $v_1 = -0.6$ in eqn (ii)

$$-(-0.6) + v_2 = 9$$

$$0.6 + v_2 = 9$$

$$v_2 = 8.4V$$

$v_1 = -0.6V$ and $v_2 = 8.4V$

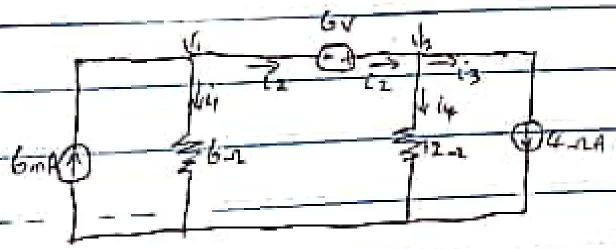
Current through the 3Ω resistor

$$i_2 = \frac{v_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2Ω resistor

$$i_4 = \frac{v_2}{4} = \frac{8.4}{4} = 4.2A$$

4)



Find the node voltage and the currents through the 6Ω and 12Ω resistor

let assume that $v_1 - v_2 = 6v \Rightarrow i_2$

At node 1, using KCL

$$6mA = i_1 + i_2$$

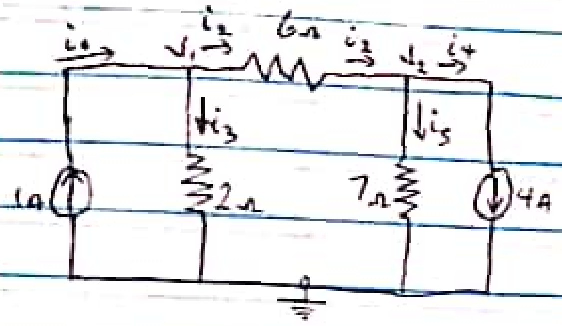
$$6mA = \frac{v_1 - 0}{6} + (v_1 - v_2)$$

$$36 = v_1 + 6(v_1 - v_2)$$

$$36 = v_1 + 6v_1 - 6v_2$$

$$36 = 7v_1 - 6v_2 \quad \text{---} (1)$$

ii) Obtain V_1 and V_2 and the currents through the motors for the circuit in example ii) if the 2A current ~~[illegible]~~ source was replaced by a 1A current source.



At node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \quad ---(1)$$

At node 2

$$i_2 = i_4 + i_5$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \quad ---(ii)$$

from eqn (i) $V_2 = 4V_1 - 6$

Substitute $V_2 = 4V_1 - 6$ in eqn (ii)

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45}$$

$$V_1 = -2V$$

Substitute $V_1 = -2$ in eqn (1)

$$6 = 4(-2) - V_2$$

Using Cramer's rule

$$5v_1 - 2v_2 - 3v_3 = 60 \quad \text{---(i)}$$
$$4v_1 - 7v_2 = 768 \quad \text{---(ii)}$$
$$-3v_1 + 4v_3 = 384 \quad \text{---(iii)}$$

In matrix representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}$$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$$= 5(-28-0) + 2(16+0) - 3(0-21)$$
$$= -140 + 32 + 63$$
$$= -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$$= 60(-28-0) - 768(-8-0) - 384(0-21)$$
$$= -1680 + 6144 - 8064$$
$$= -3600$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80\,V$$

For v_2 $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$