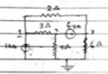
Question 1

Find the Voltages at nodes 1, 2 and 3 in the circuit below.



At node 1, KCL

$10 = i_1 + i_2 \rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 + 2V_1$ — (i)

At node 2, KCL

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2 - 0$

$765 = 4V_1 - 7V_2$ — (ii)

At Node 3, KCL

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

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$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ — (iii)

Using Crammer's Rule.

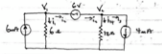
$5V_1 - 2V_2 - 3V_3 = 60$ ... (i)

$4V_1 - 7V_2 = 748$ ... (ii)

$-3V_1 + 4V_3 = 384$ ... (iii)

Question. 4

4)



Find the node Voltages and the currents through the 6Ω and 12Ω resistor.

Solution

Let say $V_1 - V_2 = 6V \rightarrow i_2$

At node 1, using KCL.

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$0.036 = V_1 + 6(V_1 - V_2)$$

$$0.036 = V_1 + 6V_1 - 6V_2$$

$$0.036 = 7V_1 - 6V_2 \quad \text{— (i)}$$

At Node 2.

$$i_2 = i_3 + i_4.$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 0.048 + V_2$$

$$0.048 = 12V_1 - 12V_2 - V_2.$$

$$0.048 = 12V_1 - 13V_2 \quad \text{— (ii)}$$

$$7V_1 - 6V_2 = 0.036 \quad \times 12 \quad \text{— i}$$

$$12V_1 - 13V_2 = 0.048 \quad \times 7 \quad \text{—}$$

$$84V_1 - 72V_2 = 0.432 \quad \text{— (iii)}$$

$$84V_1 - 91V_2 = 0.336 \quad \text{— (iv)}$$

$$0 + 19V_2 = 0.096.$$

$$V_2 = 5.053 \times 10^{-3} V$$

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Sub V_2 in eqn (iv)

$$84V_1 - 0.4598 = 0.336$$

$$84V_1 = 0.336 + 0.4598$$

$$V_1 = \frac{0.7958}{84} = 9.474 \times 10^{-3} V$$

∴ Current through the 6Ω resistor:

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \text{—(i)}$$

Using KVL for loop 2

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \text{—(ii)}$$

$$7V_1 + 8V_2 = 63 \quad \text{— } \times 1$$

$$-V_1 + V_2 = 9 \quad \text{— } \times 7$$

$$7V_1 + 8V_2 = 63 \quad \text{—(iii)}$$

$$+ \; -7V_1 + 7V_2 = 63 \quad \text{—(iv)}$$

$$0 + 15V_2 = 126$$

$$V_2 = \frac{126}{15} = 8.4V$$

Sub V_2 in eqn (iv)

$$-7V_1 + 7(8.4) = 63$$

$$-7V_1 = 63 - 58.8$$

$$-7V_1 = 4.2$$

$$V_1 = -0.6V.$$

Current through the 2 Ω resistor.

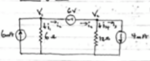
$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2 Ω resistor.

$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 2.1A.$$

Question. 4

4)



Find the node Voltages and the currents through the 6 Ω and 12 Ω resistor.

Sub $V_1 = -2$ in eqn (i)

~~6 = 4V₁~~

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$= -14V.$

$\therefore V_1 = -2V, \; V_2 = -14V.$

Current through the resistors.

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A.$

Question 3

3) 

Find the Current through the 3Ω and 2Ω resistors.

Solution

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0$ — (i)

Using KVL for loop 1

$384 = V_1 - 3(V_1 - V_3)$
$384 = -3V_1 + 4V_3$ —— (iii)

Using Crammer's Rule.

$5V_1 - 2V_2 - 3V_3 = 60$ --- (i)
$4V_1 - 7V_2 = 748$ --- (ii)
$-3V_1 + 4V_3 = 384$ --- (iii)

In Matrix Representation.

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 748 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$, $V_3 = \frac{\Delta_3}{\Delta}$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-28 - 0) + 2(16 + 0) - 3(0 - 21)$
$= -140 + 32 + 63$
$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$

$= 60(-28 - 0) - 768(-8 - 0) - 384(0$
$= -1680 + 6144 - 8064.$
$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

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$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5(768 - 0) - 4(240 + 1152) - 3(0 +$
$2244)$

At Node 1, KCL :

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_3}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ —— (i)

At Node 2 :

$i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ —— (ii)

Using Elimination Method :

$120 = -8V_1 + 5V_2 \quad \times 2$

$144 = -4V_1 + 6V_2 \quad \times 4$

$240 = -16V_1 + 10V_2$ —— (iii)

$576 \quad -16V_1 + 24V_2$ —— (iv)

$816 \quad 0 + 34V_2$

$V_2 = 24V$

Sub V_2 in eqn (iii)

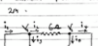
$240 = -16V_1 + 240$

$V_1 = 0$

$\therefore V_1 = 0V, \quad V_2 = 24V$

$i_1 = 0A, \; i_2 = 0A, \; i_3 = 6A, \; i_4 = -12A.$

2ii Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) & the 2A current source was replaced by a 1A current 2A.



$12V_1 - 13V_2 = 0.048 \quad \times 7$

$84V_1 - 72V_2 = 0.432$ — (vii)
$84V_1 - 91V_2 = 0.336$ — (iv)

$0 + 19V_2 = 0.096$

$V_2 = 5.053 \times 10^{-3}\,V$

Sub V_2 in eqn (iv)

$84V_1 - 0.4598 = 0.336$

$84V_1 = 0.336 + 0.4598$

$V_1 = \frac{0.7958}{84} = 9.474 \times 10^{-3}\,V$

∴ Current through the 6 Ω resistor:

$i_1 = \frac{V_1}{6} = \frac{9.474 \times 10^{-3}}{6} = 1.579 \times 10^{-3}\,A$

Current through the 12 Ω resistor:

$i_4 = \frac{V_2}{12} = \frac{5.053 \times 10^{-3}}{12} = 4.212 \times 10^{-4}\,A$

$V_1 = 9.474 \times 10^{-3}\,V$
$V_2 = 5.053 \times 10^{-3}\,V$
$i_1 = 1.579 \times 10^{-3}\,A$
$i_4 = 4.212 \times 10^{-4}\,A$.

$$V_2 = \Delta_2 = \begin{array}{c|ccc|} + & 5 & 60 & -3 \\ - & 4 & 768 & 0 \\ + & -3 & 384 & 4 \end{array}$$

$$= 5(768 - 0) - 4(240 + 1152) - 3(0 + 2244)$$

$$= 2880$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

For V_3,

$$\begin{array}{c|ccc|} + & 5 & -2 & 60 \\ - & 4 & -7 & 768 \\ + & -3 & 0 & 384 \end{array}$$

$$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$$

$$= -7020$$

$$V_3 = \frac{\Delta_3}{D} = \frac{-7020}{-45} = 156V$$

$\therefore V_1 = 80V, V_2 = -64V, V_3 = 156V$

Question 2

2. Find the voltages at nodes 1 and 2 and determine the current flowing through the four resistors in the circuit below.

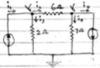
At Node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

2ii Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current 2A.



Solution

At Node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = (V_1 - V_2) + 3V_1$

$6 = 4V_1 - V_2$ --- (i)

At Node 2:

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ — (ii)

From eqn (i), $V_2 = 4V_1 - 6$.

Sub V_2 in eqn (ii).

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$168 - 78 = 7V_1 - 52V_1$

$90 = -45V_1$

$V_1 = -2V$.

Sub $V_1 = -2$ in eqn (i)

~~$6 = 4V_1$~~

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$= -14V$.