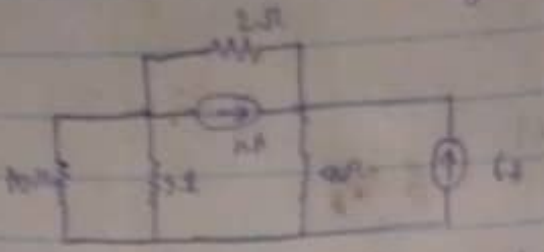
17/ENG/02/022
Computer Engineering

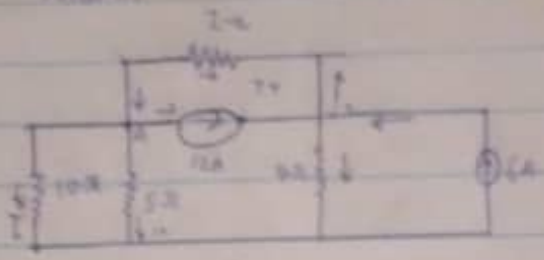
Circuit Theory

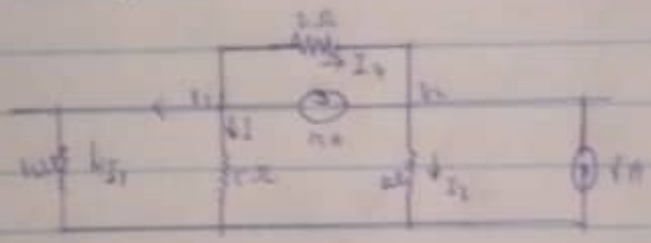
1)



i. Find the voltage at node 1 and 2
ii. Find the current flowing through the 5Ω resistor

Solution





From node 1 using KCL

$I_1 + I_2 + I_3 = -12A$

$I_1 = \frac{V_1 - 0}{10} = \frac{V_1}{10}$

$I_{5\Omega} = \frac{V_1 - 0}{5} = \frac{V_1}{5}$

$I_2 = \frac{V_1 - V_2}{2}$

$\frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -12$

$\frac{V_1}{10} + \frac{2V_1}{10} + \frac{5(V_1 - V_2)}{10} = -12$

$\frac{V_1 + 2V_1 + 5V_1 - 5V_2}{10} = -12$

$\frac{8V_1 - 5V_2}{10} = -12$

$\Rightarrow 8V_1 - 5V_2 = -120$

Applying KCL in node 2

$I_4 + I_5 + I_2 = 3$

$I_4 + I_3 = -1A$

$I_4 = \frac{V_1 - V_2}{2}$ and $I_3 = \frac{V_2 - 0}{4} = \frac{V_2}{4}$

$\Rightarrow \frac{V_1 - V_2}{2} + \frac{V_2}{4} = 1A$

$\frac{2(V_1 - V_2)}{4} + \frac{V_2}{4} = -1A$

$\frac{2V_1 - 2V_2 - V_2}{4} = -1A$

$\frac{2V_1 - 3V_2}{4} = -1A$

$2V_1 - 3V_2 = -7 \quad \ldots\ldots (1)$

$8V_1 - 6V_2 = -14$

$2V_1 - 3V_2 = -7$

using Calculator to solve simultaneously

$V_1 = 0V$ & $V_2 = 2A$

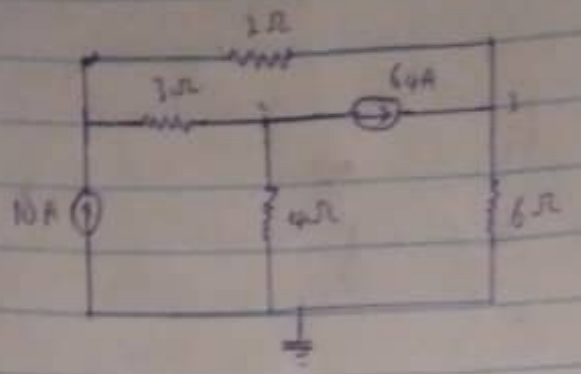
$I_1 = \frac{V_1}{10} = \frac{0}{10} = 0A$

$I_2 = \frac{V_2}{5} = \frac{0}{5} = 0A$

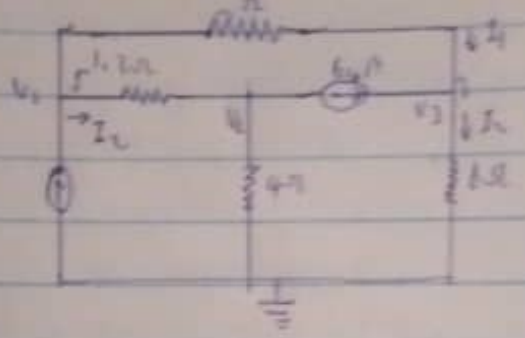
$I_3 = \frac{V_2}{4} = \frac{2A}{4} = 0.5A$

$I_4 = \frac{V_1 - V_2}{2} = \frac{0 - 2A}{2} = -1A$

2.)



Find the voltage at node 1 & 2 using solution



at node 1: using KCL $10 = I_1 + I_2$

$$10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3}$$

at node 2; using KCL: $I_2 = 6A + I_3$

$$\frac{V_1 - V_2}{3} = 6 + \frac{V_2}{4}$$

at node 3 using KCL; $I_4 = 6A + I_3$

$$\frac{V_3}{6} = 6 + \frac{V_2 - V_3}{2}$$

$$10 = \frac{5}{6}V_1 - \frac{1}{3}V_2 - \frac{1}{2}V_3$$

$$6A = \frac{1}{3}V_1 - \frac{7}{12}V_2 + 0V_3$$

$$6A = -\frac{1}{2}V_1 - 0 + \frac{1}{2}V_3$$

using matrix method

$$\begin{bmatrix} 10 \\ 6A \\ 6A \end{bmatrix} = \begin{bmatrix} \frac{5}{6} & -\frac{1}{3} & -\frac{1}{2} \\ \frac{1}{3} & -\frac{7}{12} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

A B C

$I_1 = 11A$

$$B^{-1}R = \begin{bmatrix} 3.73 & -2.87 & 2.8 \\ 2.13 & -2.93 & 1.6 \\ 2.8 & -1.6 & 3.6 \end{bmatrix} \begin{bmatrix} 16 \\ 64 \\ 64 \end{bmatrix}$$

$$C = \begin{bmatrix} 80 \\ -64 \\ 156 \end{bmatrix}$$

1. $V_1 = 80V, \quad V_2 = -64V \quad V_3 = 156V$