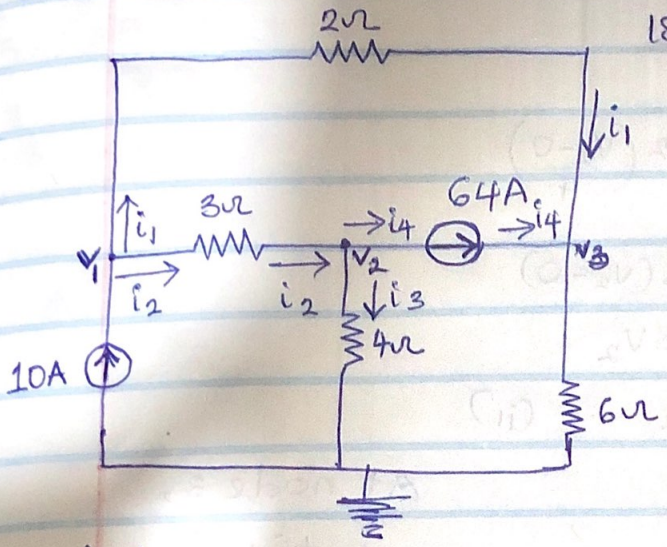
NODAL ANALYSIS EEE 322

ASSIGNMENT

ADEROJU OLUWARIKEMI ADEBOSOLA COMPUTER ENG

18/ENG02/102



At node 1,

$10 = i_1 + i_2$

$10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$\frac{3(V_1 - V_2) + 2(V_1 - V_2)}{6}$

$10 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$10 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

~~$10 = 5V_1 - 5V_2$ ...~~

$10 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$10 = 5V_1 - 2V_2 - 3V_3$ ... (1)

At node 2, ~~$i_2 = i_3 + i_4$~~

$i_2 = i_3 + 64$; $64 = i_2 - i_3$

$$64 = \frac{V_1 - V_2}{3} + \frac{V_2 - 0}{4}$$

$$64 \times 12 = 12\frac{(V_1 - V_2)}{3} + 12\left(\frac{V_2 - 0}{4}\right)$$

$$768 = 4(V_1 - V_2) + 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 + 3V_2$$

$$768 = 4V_1 - 7V_2 \quad \cdots (i)$$

At node 3,

$i_1 + i_4 = i_3$

$$\frac{V_1 - V_3}{2} + 64 = \frac{V_2 - 0}{4}$$

$$\frac{V_1 - V_3}{2} - \frac{V_2 - 0}{4} = -64$$

divide although by 2

$$\frac{\frac{V_1 - V_3}{1} - \frac{V_2 - 0}{2} = -32}{2}$$

At node 3,

$i_1 + i_4 = i_3$

$$\frac{V_1 - V_3}{2} + 64 = \frac{V_3 - 0}{6}$$

$$\frac{V_1 - V_3}{2} - \frac{V_3 - 0}{6} = -64$$

$$\frac{V_1 - V_3}{2} - \frac{V_3}{6} = -64$$

multiply through by 6

$$\left(\frac{V_1 - V_3}{2}\right)3 - \left(\frac{V_3}{6}\right)6 = -64 \times 6$$

$$(V_1 - V_3)3 - V_3 = -384$$

$$3V_1 - 3V_3 + 384 = V_2$$

$$3V_1 - 4V_3 = 384$$

$$3V_1 - 4V_3 = 384;\ 4V_3 - 3V_1 = 384 \quad \cdots (iii)$$

Using matrix rule

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$\Delta = -45$

$$\Delta_1 = \begin{vmatrix} + & - & + \\ 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60(-28) + 2(3072) - 3(2688)$

$\Delta_1 = -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

Therefore; $V_1 = 80V$

$V_2 = -64V$

$V_3 = 156V$

$$\Delta_{22} = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ 3 & 384 & 4 \end{vmatrix}$$

$= 5(3072) - 60(16) - 3(3840)$

$\Delta_{22} = 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$$\Delta_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ 3 & 0 & 384 \end{vmatrix}$$

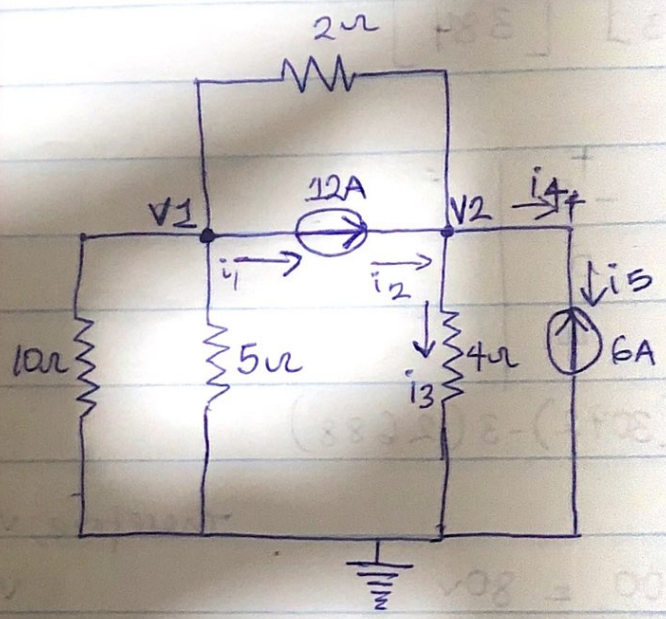
$= 5(-2688) + 2(3840) - 60(-21)$

$\Delta_3 = -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156$

Q

i) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below.



At node 1

$$\frac{-V_1}{10} - \frac{V_1}{5} = 12A + \frac{V_1 - V_2}{2}$$

$$\frac{-V_1 + 2V_1}{10} = 12A + \frac{V_1 - V_2}{2}$$

$$10\left(\frac{-V_1 + 2V_1}{10}\right) = 10(12A) + 10\left(\frac{V_1 - V_2}{2}\right)$$

$$V_1 + 2V_1 - 5V_1 + 5V_2 = 120A$$

$$-6V_1 + 7V_1 =$$

$$i_1 = i_2 + i_3 + i_4$$

$$10\left(\frac{V_0 - V_1}{10}\right) = \left(\frac{V_1 - V_2}{2}\right)10^5 + 120 + \left(\frac{V_1 - V_3}{5}\right)10^2$$

$$V_0 - V_1 = 5V_1 - 5V_2 + 120 + 2V_1 - 2V_2$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \quad \text{--- (i)}$$

Node 2,

$$i_3 + i_2 + i_5 = i_3$$

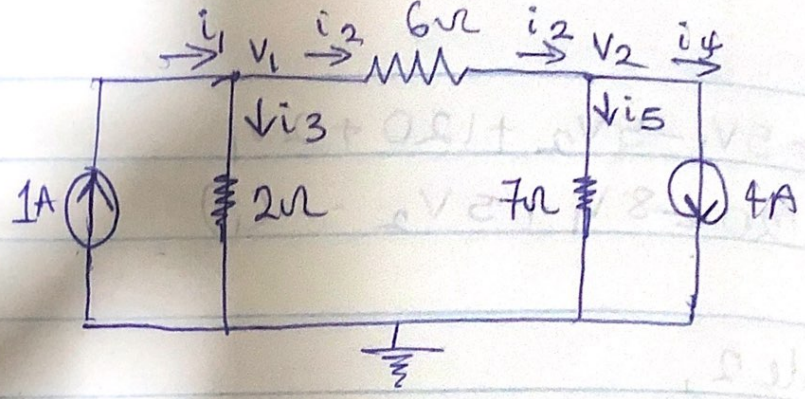
$$\frac{12 + V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$\frac{12 + V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

multiply through by 8

$$(12 \times 8) + \left(\frac{V_1 - V_2}{2}\right) \times 8 + (6 \times 8) = \left(\frac{V_2 - 0}{4}\right) \times 8$$

Class-work



Node 1

$i_1 = i_2 + i_3$

$\frac{1}{1} = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ -- (i)

Node 2

$i_2 = i_4 + i_5$

$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ -- (2)

$V_2 = 4V_1 - 6$ --- (3)

Substitute V_2 into eqn(2)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$168 - 78 = -45V_1$

$90 = -45V_1$

$V_1 = -2V$

$V_2 = 4(-2) - 6 = -14V$

Therefore, the current accross the resistor

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 - -14}{6} = \frac{12}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2V}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$