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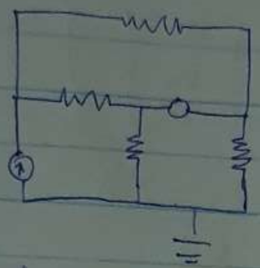
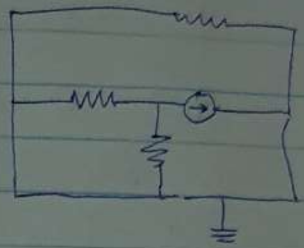
17/ENG02/020
COMPUTER ENGINEERING

1) Find the voltages at nodes 1, 2 and 3 in the circuit below





At node 1 kcl:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_2 \text{ ---- (i)}$$

At node 2 kcl:

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - \cancel{0}}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \text{ ---- (ii)}$$

At node 3, kcl

$$64 + i = i_5$$

$$64 = i_5 - i$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_2}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$384 = -3V_1 + 4V_3$ - - - (iii)

using Cramer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ - - - (i)

$4V_1 - 7V_2 = 768$ - - - (ii)

$-3V_1 + 4V_3 = 384$ - - - (iii)

in matrix representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \Delta^1/\Delta$, $V_2 = \Delta^2/\Delta$, $V_3 = \Delta^3/\Delta$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$

$= 5(-280) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$\Delta = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) - 384(-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$V_1 = \Delta^1/\Delta = -3600/-45 = 80V$

for V_2: $\Delta_2 = \begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 \cdot -3))$

$- 3(0 - (768 \cdot -3))$

$= 2880$

$\therefore V_2 = \Delta^2/\Delta = 2880/-45 = -64V$

for V_3; $\begin{array}{c} + \\ - \\ + \end{array}\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

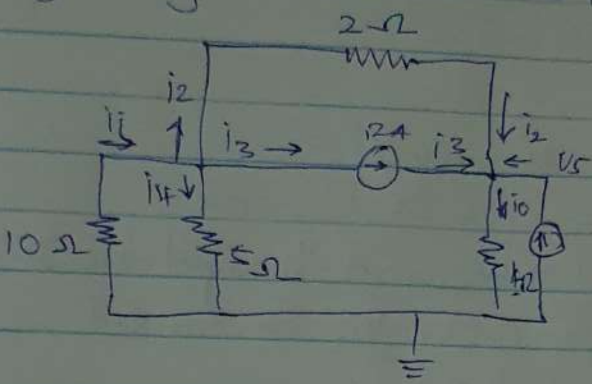
$= 5((-7\times 384)-0) - 4((-2\times 384-0) - 3((-2\times 768)-(-2\times 60)$

$= -7020$

$\therefore V_3 = \Delta_3/\Delta = -7020/-45 = 156V$

Hence $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

2) i) find the voltages at nodes 1 and the currents flowing through the four resistors in the circuit below



at node 1; KCL

$i_1 = i_2 + i_3 + i_4$

$V_0 - V_1/10 = V_1 - V_2/2 + 12 + V_1 - V_1/5$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ --- (i)

At Node 2

$i_2 + i_2 + i_5 = i$

$12 + V_1 - V_2/2 + 0 = V_2 - 0/4$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_i + 6V_2$ --- (ii)

using Elimination method

$120 = -8V_1 + 5V_2$ --- (i) $\times -4$

$144 = -4V_1 + 6V_2$ --- (ii) $\times -8$

$-480 = 32V_1 - 20V_2$ ---- (iii)

$-1152 = 32V_1 - 48V_2$ --- (iv)

sub tract eqn (iii) from (iv)

$-672 = 0 - 28V_2$

$V_2 = -672/-28$

$V_2 = 24\,V$

Subs $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$V_1 = 144 - 6V_2/-4$

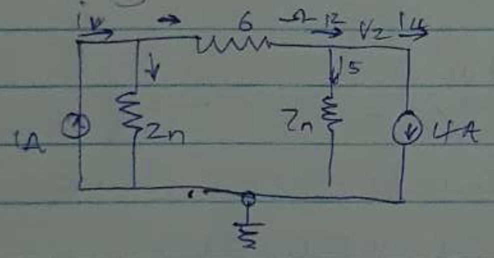
$V_1 = 144 - 6V_2/-4$

$V_1 = 0$

$\therefore V_1 = 0V,\ V_2 = 24V$

$i = 0A,\ i_2 = 0A,\ i_3 = 6A,\ i_4 = -12A$

ii) obtain V_1 and V_2 and the current throught the resistor to the circuit in example (ii) if the 2A current source is replaced by a 1a current source



at node 1

$i = i_2 + i_3$

$1 = V_1 - V_2/6 + V_2/2$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2 \quad \cdots (i)$

At node 2

$i_2 = i_4 + i_5$

$V_1 - V_2/6 = 4 + V_2/7$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2 \quad \cdots (ii)$

from eqn (i); $V_2 = 4V_1 - 6$

Subs $V_2 = 4V_1 - 6$ in eqn

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = 90/-45$

$V_1 = -2V$

Subs. $V_1 = -2$ in eqn (i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$\therefore V_1 = -2V, \; V_2 = -14V$

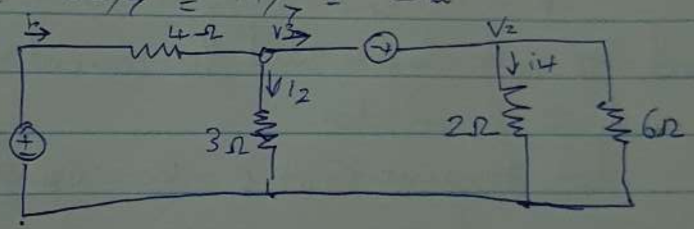
Current through the resistors

$i_2 = V_1 - V_2 / 6 = -2 + 14 / 6 = 2A$

$i_3 = V_1/2 = -2/2 = -1A$

$i_5 = V_2/7 = -14/7 = -2A$

3)



find the current through the 3Ω and 2Ω resistors

using KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$V_1 - 21/4 + V_1/3 + V_2/6 + V_2/2$

$7V_1 + 8V_2 - 63 = 0 \quad \ldots (i)$

using KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9 \quad \ldots (ii)$

$7V_1 + 8V_2 = 63 \quad \ldots (i)$

$-V_1 + V_2 = 9 \quad \ldots (ii)$

let $v_2 = 9 + v_1$

Sub $v_2 = 9 + v_1$ in eqn(i)

$7v_1 + 8(9 + v_1) = 63$

$7v_1 + 72 + 8v_1 = 63$

$15v_1 = -9$

$v_1 = -0.6V$

Sub $v_1 = -0.6$ in eqn (ii)

$-(-0.6) + v_2 = 9$

$0.6 + v_2 = 9$

$v_2 = 8.4V$

$\therefore v_1 = -0.6V$ and $v_2 = 8.4V$

Current through the 3 Ω resistors;

$i_2 = v_1/3 = -0.6/3 = -0.2A$

Current through the 2 Ω resistor

$i_4 = v_2/4 = 8.4/4 = 4.2A$

find the node voltages and the currents through the 6 Ω and 12 Ω resistor

let Assume that $v_1 - v_2 = 6\Omega \Rightarrow i_2$

At node 1; using KCL

$6mA = i_1 + i_2$

$6mA = v_1 - 0/6 + (v_1 - v_2)$

$36 = v_1 + 6(v_1 - v_2)$

$36 = v_1 + 6v_1 - 6v_2$

$36 = 7v_1 - 6v_2$ --- (i)

At Node 2

$i_2 = i_3 + i_4$

$v_1 - v_2 = 4mA + v_2 - 0/12$

$12(v_1 - v_2) = 48 + v_2$

$48 = 12v_1 - 12v_2 - v_2$

$48 = 12V - 13v_2$ --- (ii)

Solving V_1 and V_2 simultaneously

$V_1 = 9.5V$ and $V_2 = 5.1V$

$\therefore$ Current through the 12Ω resistor

$i_4 = V_2/12 = 5.1/12 = 0.43A$

$\therefore V_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_4 = 0.43A$