

$$(1) \frac{(3x-1) dx}{(x-1)(x-2)(x-3)}$$

$$(2) \frac{(x^2+x+1) dx}{(x+2)(x^2+1)}$$

$$(3) \frac{(x^2+1)}{(x-1)(x-2)^2}$$

$$(4) \frac{(x^3+x^2+x+1) dx}{(x+1)}$$

Solo

$$(1) \frac{(3x-1)}{(x-1)(x-2)(x-3)}$$

$$\frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)} = \frac{3x-1}{(x-1)(x-2)(x-3)}$$

$$A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = 3x-1$$

$$A(x^2-3x-2x+6) + B(x^2-3x-x+3) + C(x^2-2x-x+2) = 3x-1$$

$$A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2) = 3x-1$$

$$Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C = 3x - 1$$

Multiply B by $(x^2+1)(x+2)$

$$(Ax^2+Bx+C)(x+2) + D(x^2+1) = (x^2+x+1)$$

$$Ax^3 + Ax^2 + Bx^2 + 2Bx + Cx + 2C + Dx^2 + D = (x^2+x+1)$$

$$(A)x^3 + (A+B) x^2 + (2B+C)x + (2C+D) = (x^2+x+1)$$

$$(A) A = 0 \quad \text{--- (1)}$$

$$2A + B + D = 1 \quad \text{--- (2)}$$

$$2B + C = 1 \quad \text{--- (3)}$$

$$2C + D = 1 \quad \text{--- (4)}$$

Put $A=0$ in (2)

$$2(0) + B + D = 1$$

$$B + D = 1$$

$$B = -D$$

Put $B = -D$ in (3)

$$2(-D) + C = 1$$

$$-2D + C = 1 \quad \text{--- (5)}$$

$$C - 2D = 1 \quad \text{--- (6)}$$

By elimination method

$$1 \times 2C + D = 1$$

$$2 \times C - 2D = 1$$

$$2C + D = 1$$

$$-2C - 4D = 2$$

$$-3D = -1$$

$$D = \frac{-1}{-3}$$

$$D = \frac{1}{3}$$

$$C - 2D = 1$$

$$C - 2\left(\frac{1}{3}\right) = 1$$

$$a - \frac{2}{3} = 1$$

$$c = 1 - \frac{2}{3}$$

$$c = \frac{1}{3}$$

$$B = -D$$

$$B = -\left(\frac{1}{3}\right)$$

$$B = -\frac{1}{3}$$

$$A = 0, B = -\frac{1}{3}, C = \frac{1}{3}, D = \frac{1}{3}$$

$$\int \frac{0 \, dx}{(x^2+1)} + \frac{\left(-\frac{1}{3}\right) dx}{x^2+1} + \frac{\left(\frac{1}{3}\right) dx}{x^2+1} + \frac{1}{3} dx$$

$$\Rightarrow -\frac{1}{3} \ln(x^2+1) + \frac{1}{3} \ln(x^2+1) - \frac{1}{3} \ln(x+2)$$

$$(3) \frac{(x^2+1) \, dx}{(x-3)(x-2)^2}$$

$$\left(\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \right)$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

$$B = \frac{5}{3}$$

$$B = -\frac{5}{3}$$

$$A + B + C = 0$$

$$A + \left(-\frac{5}{3}\right) + \frac{3}{2} = 0$$

$$A - \frac{1}{6} = 0$$

$$A = \frac{1}{6}$$

$$\int \frac{\frac{1}{6} dx}{x-1} + \frac{\left(-\frac{5}{3}\right) dx}{x-2} + \frac{\frac{3}{2} dx}{x-3}$$

$$\frac{1}{6} \ln|x-1| - \frac{5}{3} \ln|x-2| + \frac{3}{2} \ln|x-3| + C$$

$$\textcircled{2} \int \frac{(x^2+x+1) dx}{(x+2)(x^2+1)} \quad \int \frac{(x^2+x+1) dx}{(x^2+1)(x+2)}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+Bx+C}{x+2} + \frac{D}{x^2+1} = \frac{(x^2+x+1)}{(x+2)(x^2+1)}$$

$$(Ax^2+Bx+C)(x^2+1) + D(x+2)$$

$$\frac{Ax^2+Bx+C}{x^2+2} + \frac{D}{x+2} = \frac{(x^2+x+1)}{(x^2+1)(x+2)}$$

$$(Ax^2+Bx+C)(x+2) + D(x^2+1) = (x^2+x+1)(x+2)$$

$$2x - B + C = 4$$

$$1 + 2B - 3C = -3$$

$$-2B + 2C = 8$$

$$-2B + 3C = 3$$

$$-C = 5$$

$$C = -5$$

$$-B + C = 4$$

$$-B - 5 = 4$$

$$-B = 4 + 5$$

$$-B = 9$$

$$B = -9$$

$$A + B = 1$$

$$A + (-9) = 1$$

$$A = 1 - (-9)$$

$$A = 10$$

$$\int \frac{(x^2 + 1)}{(x-3)(x-2)^2} dx = \int \frac{10}{(x-3)} dx + \int \frac{-9}{x-2} dx + \int \frac{-5}{(x-2)^2} dx$$

$$= 10 \ln|x-3| - 9 \ln|x-2| - \frac{5}{x-2} + C$$

$$= 10 \ln|x-3| - 9 \ln|x-2| - \frac{5}{x-2} + C$$