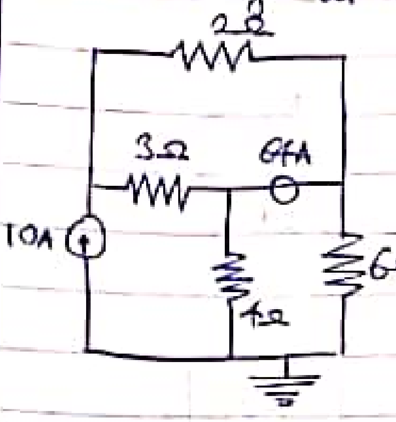
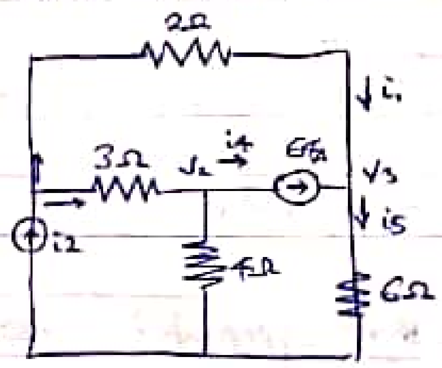
BASSEY SAVIOUR JOSEPH
16/ENG05/015
Mechatronics Engineering
EEE 322 – Electric Circuit Theory

① Find the voltages at nodes 1, 2 and 3 in the circuit below





Solution

At node 1, KCL:

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{5}$

$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 3V_1 - 2V_2 - 3V_3$ ------ (i)

At node 2, KCL:

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2$

$768 = 4V_1 - 7V_2$ -------- (ii)

At node 3, KCL:

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$$384 = -3V_1 + 4V_3 \quad \text{------ (iii)}$$

Using Cramer's Rule

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \text{---- (i)}$$

$$4V_1 - 7V_2 = 768 \quad \text{---- (ii)}$$

$$-3V_1 + 4V_3 = 384 \quad \text{---- (iii)}$$

In Matrix representation ⟹

$$\begin{pmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

where
$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix} = 5(-28-0) + 2(16+0) - 3(0-21)$$
$$= -140 + 32 + 63$$
$$= -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) + 384(0-21)$$
$$= -1680 + 6144 - 8064$$
$$= -3600$$

$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45}$$
$$= 80V$$

$$\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix} = 5(3072 - 0) - 4(240 + 1152)$$
$$-3(0 + 2304)$$

$= 15360 - 5558 - 6912$

$= 2890$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2890}{-45} = -64.22222222V$

$$V_3 = \begin{matrix} + \\ - \\ + \end{matrix}\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

$= 5(-2688 - 0) - 4(-768 - 0) - 3(~~\ldots~~ - 1536 + 420)$

$= -13440 + 3072 + 3348$

$= -7020$

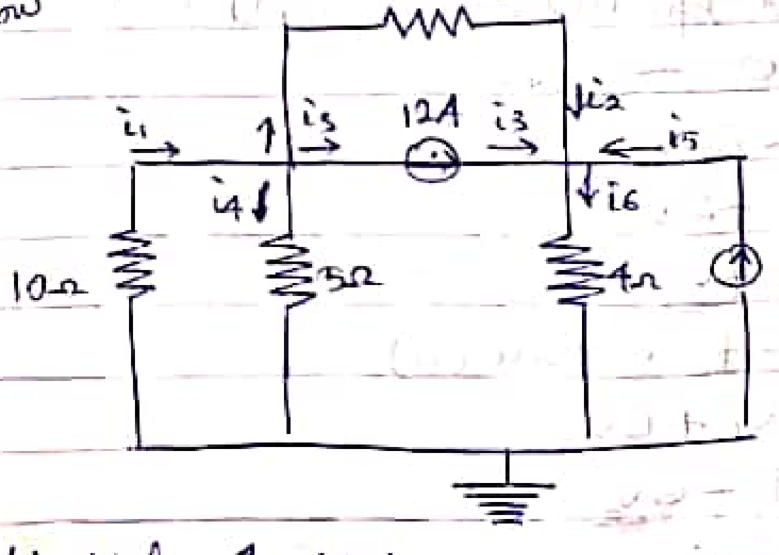
$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

Hence $V_1 = 80V$

$V_2 = -64V$

$V_3 = 156V$

(3)(5) Find the voltages at nodes 1 and 2 and determine the currents flowing through the ~~floor~~ four resistors in the circuit below



At Node 1; KCL

$i_1 = i_2 + i_3 + i_4$

$i_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$-V_1 = 7V_1 - 5V_2 + 120$

$-V_1 - 7V_1 + 5V_2 = 120$

$-8V_1 + 5V_2 = 120$ ------ (i)

At node 2

$i_3 + i_2 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ ------ (ii)

Using Elimination Method

$120 = -8V_1 + 5V_2$ ------ (i) $\times(-4)$

$144 = -4V_1 + 6V_2$ ------ (ii) $\times(-8)$

$-480 = 32V_1 - 20V_2$ ---- (iii)

$-1152 = 32V_1 - 48V_2$ ---- (iv)

Subtract equ(iii) from equ(iv)

$-672 = 0 - 28V_2$

$$V_2 = \frac{-672}{-28}$$

$\therefore V_2 = 24V$

Substitute $V_2 = 24$ in equ (ii)

$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}$$

$$V_1 = \frac{144 - 6V_2}{-4} = \frac{144 - 6(24)}{-4}$$

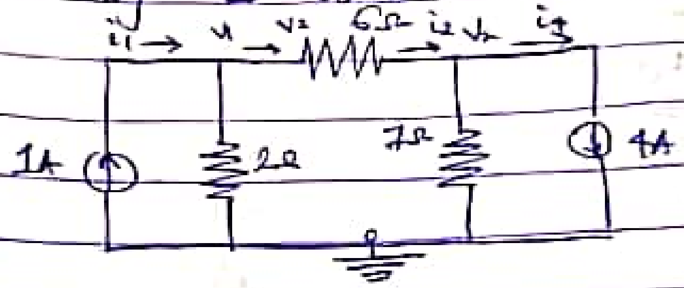
$V_1 = 0$

$\therefore V_1 = 0V$

$V_2 = 24V$

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

2 (ii) Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current source



At Node 1

$i_1 = i_2 + i_3$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ ------ (i)

At Node 2

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 7V_2 - 6V_2$

$168 = 7V_1 - 13V_2$ ------ (ii)

from equ (i), $V_2 = 4V_1 - 6$

Substitute $V_2 = 4V_1 - 6$ in equ (ii)

$168 = 7V_1 - 7(4V_1 - 6) - 6(4V_1 - 6)$

$168 = 7V_1 - 28V_1 + 42 - 24V_1 + 36$

$168 = -45V_1 + 78$

$45V_1 = -168 + 78$

$45V_1 = -90$

$V_1 = \frac{-90}{45}$

$V_1 = -2V$

Substitute $V_1 = -2$ in equ(i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

$\therefore V_1 = -2V$

$V_2 = -14V$

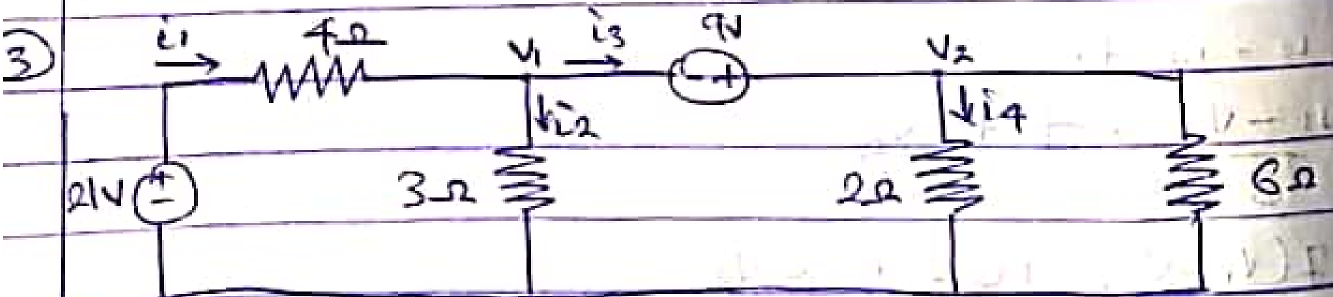
Current through the resistors;

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

3)



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2} + \frac{V_2}{2}$$

$7V_1 + 8V_2 - 63 = 0$ ---- (i)

using KVL for loop 1

$-V_1 - 9 + V_2 = 0$ ---- (ii)

$-V_1 + V_2 = 9$

---- (i)

$7V_1 + 8V_2 = 63$ ---- (i)

$-V_1 + V_2 = 9$ ---- (ii)

In equ (ii) let $V_2 = 9 + V_1$

Substitute $V_2 = 9 + V_1$ in equ (i)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = 63 - 72$

$15V_1 = -9$

$V_1 = \frac{-9}{15} = -0.6V$

$V_1 = -0.6V$

Substitute $V_1 = -0.6V$ into equ (ii)

$+0.6 + V_2 = 9$

$V_2 = 9 - 0.6$

$V_2 = 8.4V$

∴ $V_1 = -0.6V$

$V_2 = 8.4V$

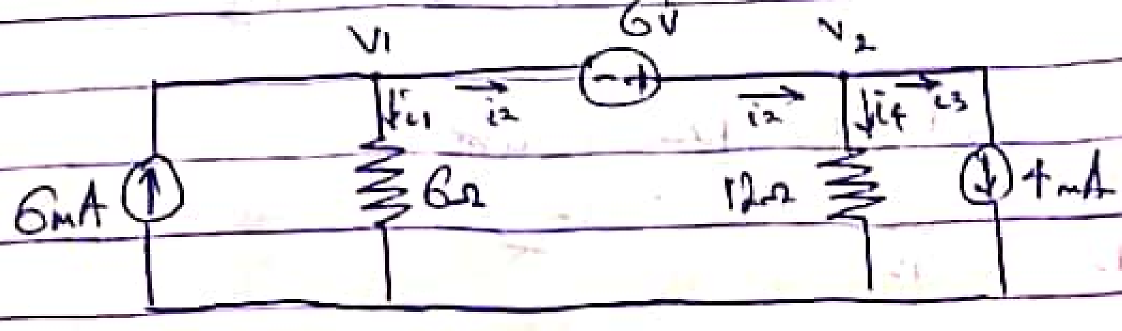
- Current through the 3Ω resistor;

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

- Current through the 4Ω resistor;

$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 2.1A$

(4)



Find the node voltages and the currents through the 6Ω and 12Ω resistor.

Let Assume that $V_1 - V_2 = 6V \Rightarrow i_2$

At node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2$ ——— (i)

At node 2.

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$12V_1 - 12V_2 = 48 + V_2$

$12V_1 - 12V_2 - V_2 = 48$

$12V_1 - 13V_2 = 48$ ——— (ii)

Solving V_1 & V_2 Simultaneously

$V_1 = 9.5V$

$V_2 = 5.1V$

∴ Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$

$i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$\therefore$ $V_1 = 9.8V$

$V_2 = 5.1V$

$i_1 = 1.58A$

$i_4 = 0.43A$