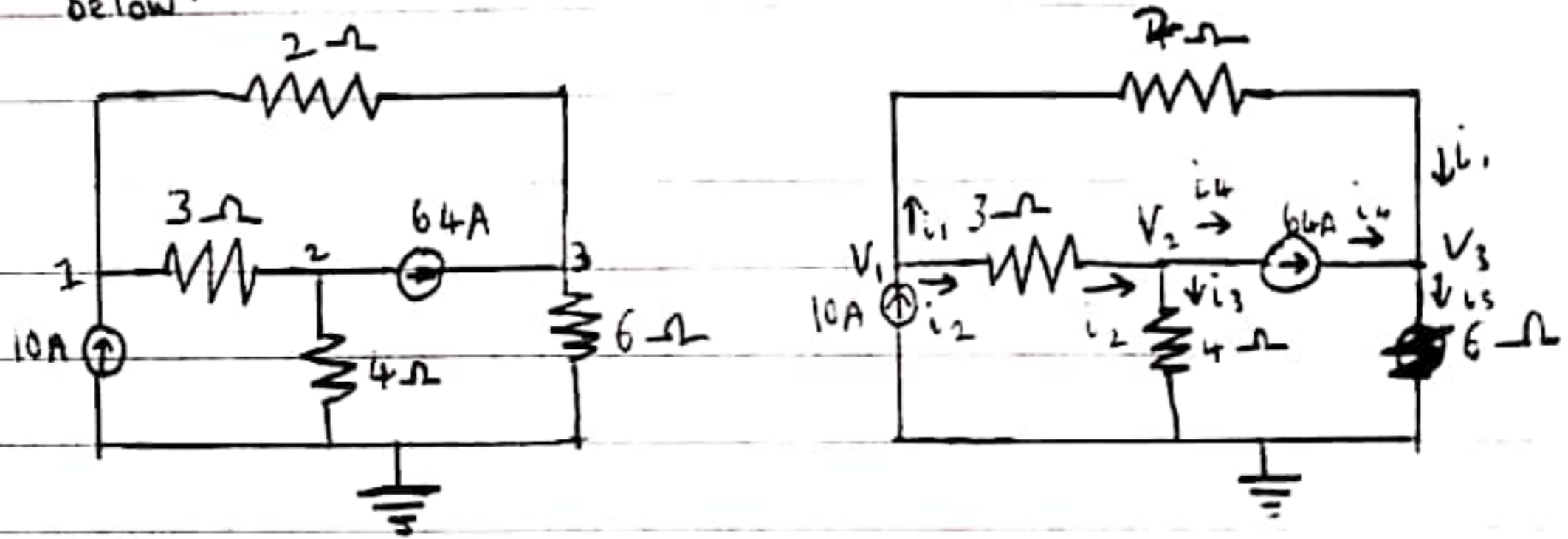


1) Find the voltages at nodes 1, 2 and 3 in the circuit below:



At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \dots (i)$$

At Node 2, KCL:

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \dots (ii)$$

At Node 3, KCL:

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

$$384 = -3V_1 + 4V_3 \quad \dots (iii)$$

Using Cramer's Rule

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \dots (i)$$

$$4V_1 - 7V_2 = 768 \quad \dots (ii)$$

$$-3V_1 + 4V_3 = 384 \quad \dots (iii)$$

In Matrix Representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix} \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$$\begin{aligned} &= 5(-28-0) + 2(16+0) - 3(0-21) \\ &= -140 + 32 + 63 \\ &= -45 \end{aligned}$$

$$\begin{aligned} \Delta_1 &= \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) + 384(0-21) \\ &= -1680 + 6144 - 8064 \\ &= -3600 \end{aligned}$$

$$\therefore V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80 \text{ V}$$

For V_2 : $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$$= 60(8-0) - 768(20-9) + 384(0+12)$$

$$= 480 - 8448 + 4608$$

$$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - (768 \times -3))$$

$$= 2880$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

For V_3 ;	+	5	-2	60
	-	4	-7	768
	+	-3	0	384

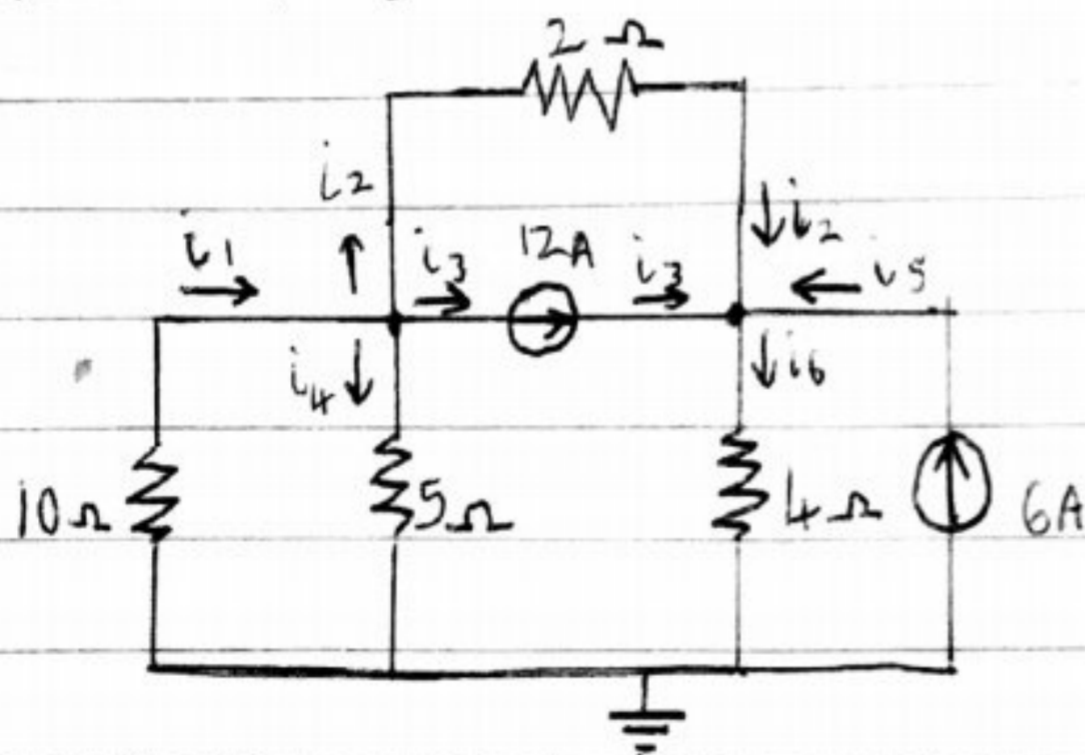
$$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$$

$$= -7020$$

$$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$$

Hence $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

2) i) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below;



At Node 1; KCL

$$i_1 = i_2 + i_3 + i_4$$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$$

$$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$$

$$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$$

$$120 = -8V_1 + 5V_2 \dots (i)$$

At Node 2

$$i_3 + i_2 + i_5 = i_6$$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$$96 + 4(V_1 - V_2) + 48 = 2(V_2)$$

$$144 + 4V_1 - 4V_2 = 2V_2$$

$$144 = -4V_1 + 6V_2 \dots (ii)$$

~~From eqn (i)~~ Using Elimination Method

$$120 = -8V_1 + 5V_2 \dots (i) \quad \times -4$$

$$144 = -4V_1 + 6V_2 \dots (ii) \quad \times -8$$

$$-480 = 32V_1 - 20V_2 \dots (iii)$$

$$-1152 = 32V_1 - 48V_2 \dots (iv)$$

Subtract eqn (iii) from (iv)

$$-672 = 0 - 28V_2$$

$$V_2 = \frac{-672}{-28}$$

$$V_2 = 24 \text{ V}$$

Subs. $V_2 = 24$ in eqn (ii)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6V_2}{-4}$$

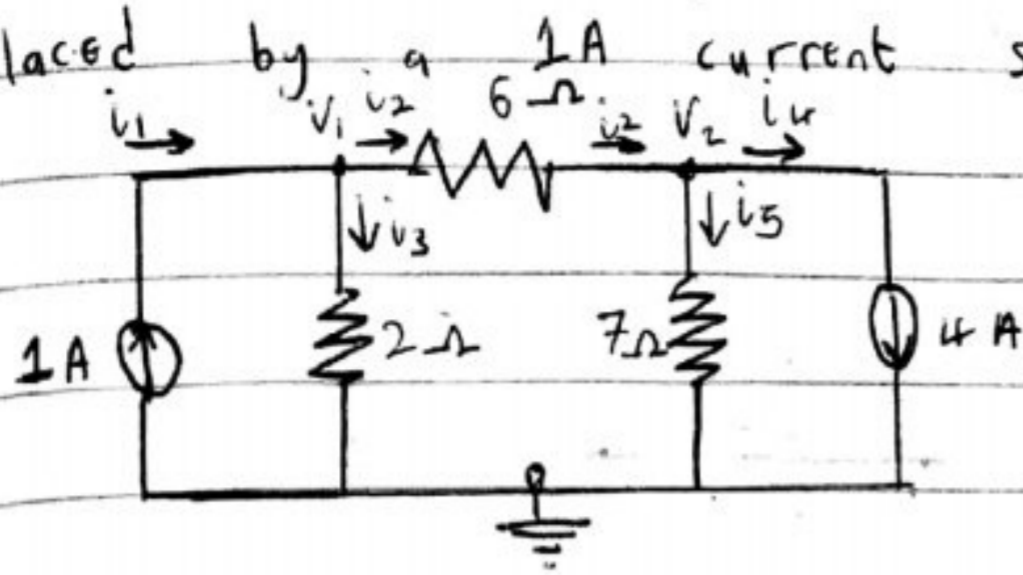
$$V_1 = \frac{144 - 6V_2}{-4}$$

$$V_1 = 0$$

$$\therefore V_1 = 0 \text{ V}, \quad V_2 = 24 \text{ V}$$

$$i_1 = 0 \text{ A}, \quad i_2 = 0 \text{ A}, \quad i_3 = 6 \text{ A}, \quad i_4 = -12 \text{ A}$$

ii) Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (ii) if the 2A current source was replaced by a 1A current source



At Node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \dots (i)$$

At Node 2

$$i_2 = i_4 + i_5$$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \dots (ii)$$

From eqn (i); $V_2 = 4V_1 - 6$

Subs. $V_2 = 4V_1 - 6$ in eqn (ii)

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45}$$

$$V_1 = -2V$$

Subs. $V_1 = -2$ in eqn (i)

$$6 = 4(-2) - V_2$$

$$6 = -8 - V_2$$

$$V_2 = -8 - 6$$

$$V_2 = -14V$$

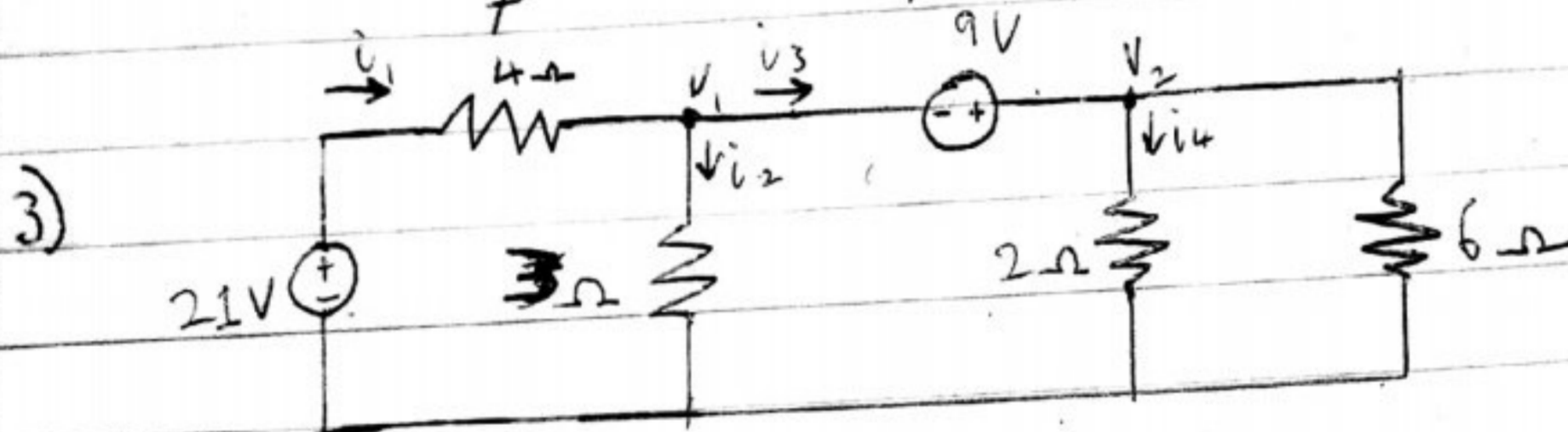
$$\therefore V_1 = -2V, \quad V_2 = -14V$$

Current through the resistors:

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = \underline{\underline{2A}}$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$$



Find the current through the 3Ω and 2Ω resistors

Using KCL at Node 1

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2} = 0$$

$$7V_1 + 8V_2 - 63 = 0 \quad \dots (i)$$

Using KVL for Loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \dots (ii)$$

$$7V_1 + 8V_2 = 63 \quad \dots (i)$$

$$-V_1 + V_2 = 9 \quad \dots (ii)$$

From (ii) Let $V_2 = 9 + V_1$

Sub $V_2 = 9 + V_1$ in eqn (i)

$$7V_1 + 8(9 + V_1) = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = -9$$

$$V_1 = -0.6V$$

Sub $V_1 = -0.6$ in eqn (ii)

$$-(-0.6) + V_2 = 9$$

$$0.6 + V_2 = 9$$

$$V_2 = 8.4V$$

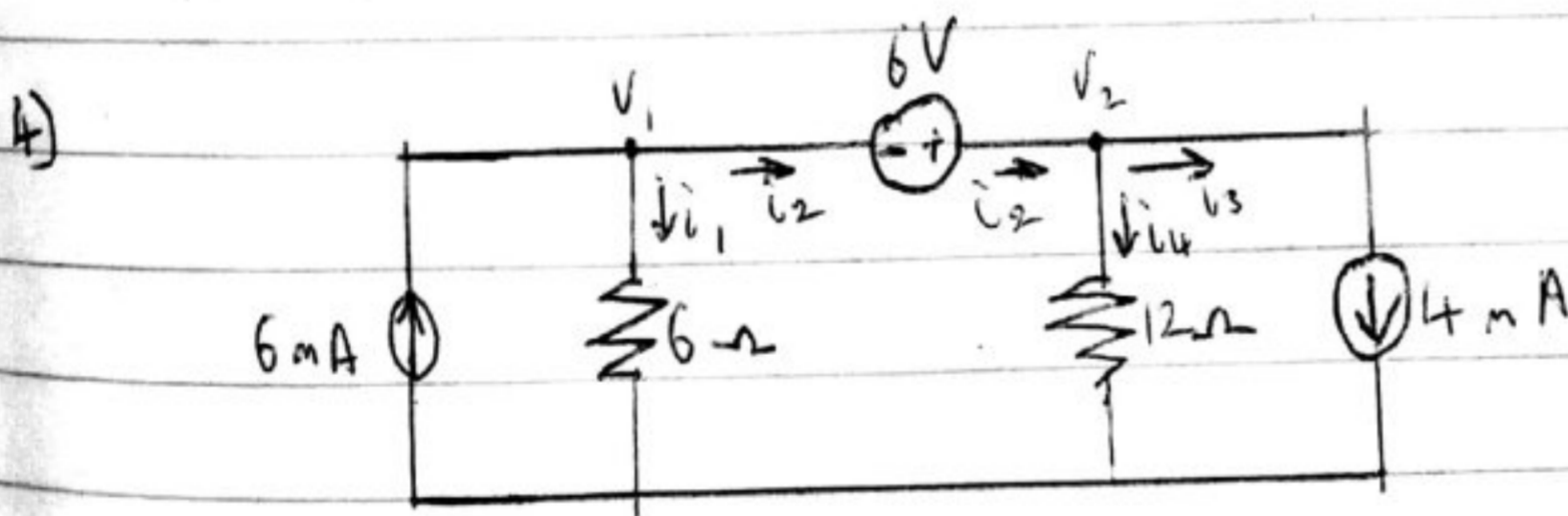
$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

Current through the 3Ω resistor;

$$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$$

Current through the 2Ω resistor

$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 2.1A$$



Find the node voltages and the currents through the 6Ω and 12Ω resistor

Let Assume that $V_1 - V_2 = 6V \Rightarrow i_2$

At node 1; using KCL

$$6 \text{ mA} = i_1 + i_2$$

$$6 \text{ mA} = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$36 = V_1 + 6(V_1 - V_2)$$

$$36 = V_1 + 6V_1 - 6V_2$$

$$36 = 7V_1 - 6V_2 \quad \dots (i)$$

At Node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4 \text{ mA} + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 48 + V_2$$

$$48 = 12V_1 - 12V_2 - V_2$$

$$48 = 12V_1 - 13V_2 \quad \dots (ii)$$

Solving V_1 and V_2 simultaneously; we have

$$V_1 = 9.5 \text{ V} \quad \text{and} \quad V_2 = 5.1 \text{ V}$$

∴ Current through the 6Ω resistor

$$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58 \text{ A}; \quad i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4 \text{ A}$$

Current through the 12Ω resistor

$$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43 \text{ A}$$

$$\therefore V_1 = 9.5 \text{ V} \quad , \quad V_2 = 5.1 \text{ V}$$

$$i_1 = 1.58 \text{ A} \quad , \quad i_4 = 0.43 \text{ A}$$