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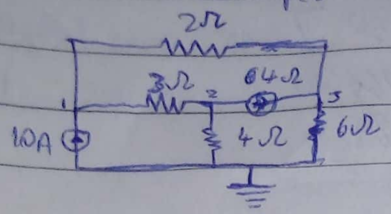
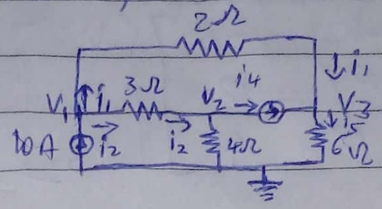
17/ENG04/020
ELECTRONICS /ELECTRICAL ENGINEERING

CIRCUIT THEORY (ENG 322) Assignment

1) Find the voltages at nodes 1, 2 and 3 in the circuit below:





At node 1, KCL:

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$

$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 2V_2 - 3V_3$ ------ (i)

At node 2, KCL:

$i_2 = i_3 + 64$

$64 = i_2 - i_3$

$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$

$768 = 4(V_1 - V_2) - 3(V_2 - 0)$

$768 = 4V_1 - 4V_2 - 3V_2 - 0$

$768 = 4V_1 - 7V_2$ ------ (ii)

At node 3, KCL:

$64 + i_1 = i_5$

$64 = i_5 - i_1$

$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$384 = V_3 - 3(V_1 - V_3)$

$384 = V_3 - 3V_1 + 3V_3$

$384 = -3V_1 + 4V_3$ ------ (iii)

Using Cramer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ ------ (1)

$$4V_1 - 7V_2 = 768 \quad \cdots (ii)$$
$$-3V_1 + 4V_3 = 384 \quad \cdots (iii)$$

In Matrix representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

Where $\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$ + − +

$$= 5(-28-0) + 2(16+0) - 3(0-21)$$
$$= -140 + 32 + 63$$
$$= -45$$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$ $= 60(-28-0) - 768(8-0) + 384(0-21)$

$$= -1680 + 6144$$
$$= -3600$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$$

for V_2 : $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$$= (5(768 \times 4) - 0) - 4((60 \times 4) - (384 \times -3)) - 3(0 - (768 \times -3))$$
$$= 2880$$

$$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$$

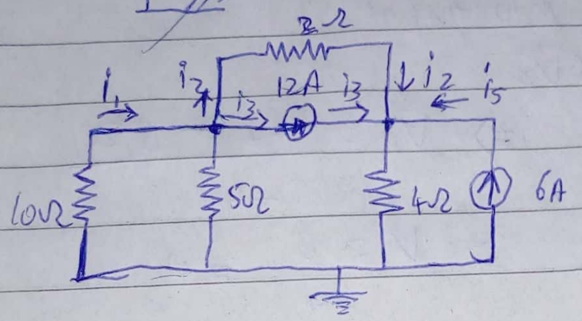
for V_3 = $\begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ 3 & 0 & 384 \end{vmatrix}$

$$= 5((-7 \times 384) - 0) - 4((-2 \times 384) - 0) - 3((-2 \times 768) - (-7 \times 60))$$
$$= -7020$$

$\therefore V_3 = \frac{D_3}{\Delta} = \frac{-7020}{-45} = 156V$

There fore, $V_1 = 80V$; $V_2 = -64V$; $V_3 = 156V$

2)(i) Find the voltages at nodes 1 and 2 and determine the currents flowing through the four resistors in the circuit below:



At Node 1, KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + i_2 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ ----- (i)

At Node 2

$i_3 + i_2 + i_5 = i_6$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2$ ----- (ii)

Making use of Elimination method.

$120 = -8V_1 + 5V_2$ ---- (i) $\times -4$

$144 = -4V_1 + 6V_2$ ---- (ii) $\times -8$

$$-480 = 32V_1 - 20V_2 \quad \text{----- (III)}$$

$$-1152 = 32V_1 - 48V_2 \quad \text{----- (IV)}$$

Subtract equ (III) from (IV)

$$-672 = 0 - 28V_2$$

$$V_2 = \frac{-672}{-28}$$

$$V_2 = 24V.$$

Substitute $V_2 = 24$ into equ (II)

$$144 = -4V_1 + 6V_2$$

$$V_1 = \frac{144 - 6V_2}{-4} \Rightarrow V_1 = \frac{144 - 6(24)}{-4}$$

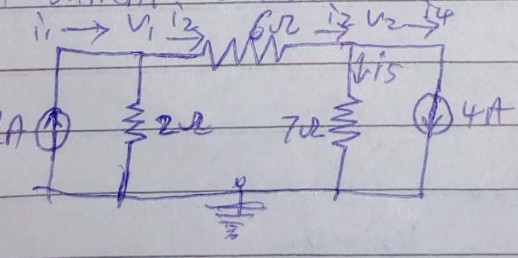
$$V_1 = \frac{144 - 144}{-4} \Rightarrow V_1 = \frac{0}{-4} = 0$$

$$\therefore V_1 = 0$$

$\therefore$ $V_1 = 0V$, $V_2 = 24V$.

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$.

(II) Obtain V_1 and V_2 and the currents through the resistors for the circuit in example (II) if the 2A current source was replaced by a 1A current source.



At Node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$$6 = V_1 - V_2 + 3V_1$$

$$6 = 4V_1 - V_2 \quad \text{----- (1)}$$

At Node 2

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$$\cancel{7(V_1 - V_2) = 6(4+V)}$$

$$7(V_1 - V_2) = 168 + 6V_2$$

$$168 = 7V_1 - 13V_2 \quad \text{....(II)}$$

from equ (I) ; $V_2 = 4V_1 - 6$

sub. $V_2 = 4V_1 - 6$ into equ (II)

$$168 = 7V_1 - 13(4V_1 - 6)$$

$$168 = 7V_1 - 52V_1 + 78$$

$$90 = -45V_1$$

$$V_1 = \frac{90}{-45}$$

$$V_1 = -2V.$$

Sub $V_1 = -2$ into equ(I)

$$6 = 4(-2) - V_2 \Rightarrow 6 = -8 - V_2$$

$$V_2 = -6 - 8$$

$$V_2 = -14V.$$

$\therefore V_1 = -2V, \; V_2 = -14V$

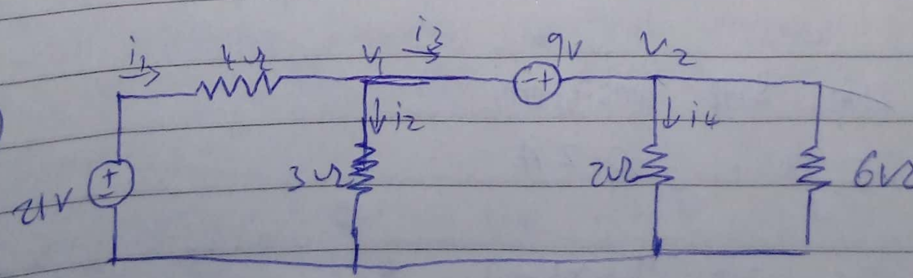
Current through the resistor

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$$

$$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$$

$$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A.$$

3)



find the current through the 3Ω and 2Ω resistors.

Using KCL at node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0$ ...... (I)

Using KVL for loop 1

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ .... (II)

$7V_1 + 8V_2 - 63 = 0$ ..... (I)

$-V_1 + V_2 = 9$ ..... (II)

from (II) let $V_2 = 9 + V_1$

Sub: $V_2 = 9 + V_1$ into equ (I)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$ $\Rightarrow V_1 = \frac{-9}{15}$

$V_1 = -0.6V$

Sub $V_1 = -0.6$ into equ (II)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 9 - 0.6$

$V_2 = 8.4V$

$\therefore V_1 = -0.6V$ & $V_2 = 8.4V$

Current through the 3Ω resistore,

$i_2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

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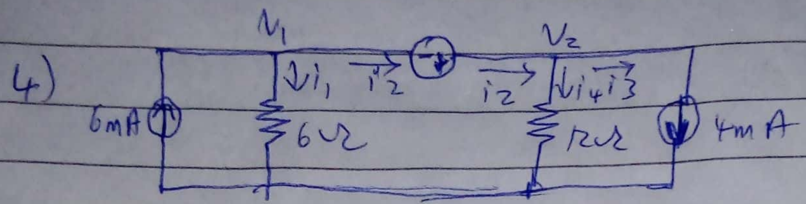
$$i_4 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2\,A$$

4)



Find the node voltages and the Currents through the 6Ω and 12Ω resistor.

Lets Assume that $V_1 - V_2 = 6V \Rightarrow i_2$

At node 1, using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$$

$$36 = V_1 + 6(V_1 - V_2)$$

$$36 = V_1 + 6V_1 - 6V_2$$

$$36 = 7V_1 - 6V_2 \quad \text{---- (I)}$$

At Node 2

$$i_2 = i_3 + i_4$$

$$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$$

$$12(V_1 - V_2) = 48 + V_2$$

$$48 = 12V_1 - 12V_2 - V_2$$

$$48 = 12V - 13V_2 \quad \text{----- (II)}$$

Solving V_1 and V_2 Simultaneously, we have

$$V_1 = 9.5V \quad \text{and } V_2 = 5.1V$$

$\therefore$ Current through the 6Ω resistor

$$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A\,; \quad i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4$$

Current through the 12Ω resistor

$$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43\,A$$

$$\therefore V_1 = 9.5V, \quad V_2 = 5.1V$$

$$i_1 = 1.58A, \quad i_4 = 0.43\,A$$